Exam Astrophysical Simulations
18 January 2022

The questions are in English, but you can reply in English or Dutch.

Question 1 (50%)

The two-step Lax-Wendroff scheme, applied to the one-dimensional continuity equation, is a combination of the Lax scheme,

$$\rho_{j,n+1} = \frac{1}{2}(1-\alpha)\rho_{j+1,n} + \frac{1}{2}(1+\alpha)\rho_{j-1,n} \tag{1}$$

and the staggered leapfrog scheme,

$$\rho_{j,n+1} = \rho_{j,n-1} - \alpha\left(\rho_{j+1,n} - \rho_{j-1,n}\right), \tag{2}$$

where we have set $\alpha = v\Delta t/\Delta x$.

(a) Use these two formulae to derive the final formula for the Lax-Wendroff scheme. Make a schematic drawing to illustrate your case.

(b) Apply a von Neumann stability analysis and derive the appropriate stability criterion.

(c) List the advantages of this integration scheme. Discuss in particular the (dis)advantages compared to the Lax scheme and to the staggered leapfrog scheme.

Question 2 (25%)

A crucial ingredient for the PM method is the Fourier convolution theorem. Formulate and prove this theorem for a 1D discrete Fourier transform,

$$\hat{x}_p = \sum_{k=0}^{K-1} x_k \, \mathrm{e}^{\frac{2\pi i k p}{K}} \qquad p = 0, 1, \ldots, K-1. \tag{3}$$

Question 3 (25%)

In the past two decades, the missing satellites problem has been one of the most disturbing problems for the ΛCDM cosmological model.

(a) Describe in two sentences what this problem is about.

(b) Explain the latest state-of-the-art: are there possible solutions from the simulation point of view ?

(c) Are there any recent developments from the observational side ?