REM - exam 25-26

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Given: formula sheet + your own cheat sheet (one double-sided A4). No calculators or Griffiths. All questions count for 5 points, and the exam counts for 60 % of your total grade. [Ter info: de puntenverdeling voor het examen en de taken is vanaf 2026-2027 anders: 80-20 in plaats van 60-40.]

On your first pass, try not to spend more than 15 minutes on each question.

0.1 Q1

From $\nabla \times \mathbf{E} = \mathbf{0}$, define the electrostatic potential and show that $\int \mathbf{E} \cdot \mathbf{dl}$ is path-independent.

0.2 Q2

A resistive cylindrical wire of length L and radius a carries a current I due to a potential difference $V = IR$ between the two ends of the cylinder. Given: hand-drawn image of the system.

- Calculate the Poynting vector in the wire. (Hint: Find the magnetic field just outside the wire, and the electric field inside the wire.)
- Compute $w = \frac{IV}{L}$.
- Interpret that last result in terms of the conservation of some physical quantity.

0.3 Q3

If magnetic charge was present, Maxwell's equations would obtain a much more symmetrical form [I copied this equation from Griffiths 5th ed., §7.3.4]:

$$\text{(i)}\ \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho_e, \qquad \text{(iii)}\ \nabla \times \mathbf{E} = -\mu_0 \mathbf{J}_m - \frac{\partial \mathbf{B}}{\partial t},$$

$$\text{(ii)}\ \nabla \cdot \mathbf{B} = \mu_0 \rho_m, \qquad \text{(iv)}\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J}_e + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

The corresponding potentials are of this form [I forgot the sign of the last term in the magnetic field]:

$$\mathbf{E} = -\nabla V - \frac{\mathrm{d}\mathbf{A}}{\mathrm{dt}} - \nabla \times \mathbf{Z}, \tag{1}$$

$$\mathbf{B} = -\nabla U + \nabla \times \mathbf{A} \pm \frac{1}{c^2}\frac{\mathrm{d}\mathbf{Z}}{\mathrm{dt}}. \tag{2}$$

- Find the Lorenz gauge equations for these potentials. (Hint: each divergence equation yields an inhomogeneous wave equation.)
- What is the equation for the curl of the electric field, in potential form?

0.4 Q4

Given: Lorentz transformations. Why is it time *dilation* but length *contraction*? Work out both of these effects from the Lorentz transformations, then discuss.

0.5 Q5

Two events A and B occur. In frame S, A causes B with an influence faster than light.

- Plot the two events on a spacetime diagram and discuss.
- Prove that there exists an inertial S' which moves at a speed less than c with respect to S, where B happens before A.

This exercise proves that influences faster than light violate causality.

0.6 Q6

Given: figure with a point A on Earth, a point B on a space station that's stationary with respect to A, and a point C which follows a spaceship going from A to B. The spaceship at C moves away from A at a speed of $v = 0.6c$. An observer at A flashes a light every 6 minutes.

- How frequent are the light flashes in point C? (Hint: the time between flashes is not 7.5 minutes; the correct answer is 12 minutes. If you got 7.5 minutes, you're missing something.)
- Suppose an observer at C sends out flashes at the same frequency as they're receiving them. How frequent are those flashes in B?

0.7 Q7

A particle with rest mass m_0, moving at a speed $v = 0.6c$, collides with a stationary particle which also has a rest mass m_0. The two particles are clumped together after the collision. What is the end rest mass and velocity of this conglomerate?

0.8 Q8

If $\mathbf{E}$ and $\mathbf{B}$ represent the fields in an inertial frame S, then the fields in a frame $\bar{S}$ are given by [I again copied this from Griffiths, §12.3.2, Lorentz x-boost]:

$$\boxed{\begin{array}{lll} \bar{E}_x = E_x, & \bar{E}_y = \gamma(E_y - vB_z), & \bar{E}_z = \gamma(E_z + vB_y), \\ \bar{B}_x = B_x, & \bar{B}_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right), & \bar{B}_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right). \end{array}}$$

Is it possible to have $\bar{\mathbf{E}} = \mathbf{0}$ in $\bar{S}$ when the fields in S are both nonzero? If so, when? Solve this problem through the following steps:

- Without loss of generality, assume that $\mathbf{E} = E_z\hat{\mathbf{z}}$ and $B_x = 0$. [This wasn't asked in the question, but it's good to think about why no generality is lost when you do this.]
- Perform a Lorentz boost in the x direction, and find constraints on the fields in S. (Hint: Lorentz boosts in other directions won't do the trick.)
- From this, find a coordinate-independent expression for $\mathbf{E}$ and $\mathbf{B}$.

0.9 Q9

You stumble upon the following weird force law:

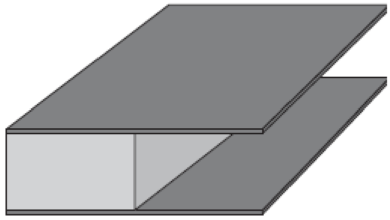
$$K^\mu = \tilde{q} \cdot \eta_\nu \cdot G^{\mu\nu}, \tag{3}$$

where $G^{\mu\nu}$ is the dual tensor.

- Compute this equation explicitly for $\mu = 1$.
- Guess the y and z components of the equation. Put this all together into one 3-vector equation.
- What could the physical interpretation of $\tilde{q}$ be?

0.10 Q10

Assume a potential difference V is applied over the half-filled parallel-plate capacitor shown below. The left half of this capacitor is a linear dielectric with a dielectric constant ε_{r}.



Find **E**, **D** and **P** in each region. Also, locate and compute all of the free and bound charges. You can ignore edge effects.

[Note: this is literally a subproblem from Griffiths, namely the second figure of problem 4.19 (p. 191 (4th ed.), p. 192 (5th ed.).]

0.11 Q11

A spherical shell of radius R has the following electrostatic potential on its surface:

$$V(R,\theta) = \frac{V_0}{2}(5\cos^3\theta - 3\cos\theta). \tag{4}$$

- Find the potential inside and outside the sphere.
- What is the electrostatic quadrupole moment?

0.12 Q12

Given: really complicated formulas for the fields, akin to:

$$\mathbf{E} = \frac{\mu_0 m_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r} + \frac{67\cos\theta}{r^2} - \frac{420k}{r^3}\right)\cos(\omega t_{\mathrm{r}})\hat{\phi}, \tag{5}$$

$$\mathbf{B} = \frac{-\mu_0 m_0 \omega^2}{4\pi c^2}\left(\frac{\sin\theta}{r} - \frac{41k}{r^2} - \frac{727\cos\theta}{r^3}\right)\cos(\omega t_{\mathrm{r}})\hat{\theta}, \tag{6}$$

Compute the total power radiated to infinity. Hint: only use terms that count for radiation. The end result that you should obtain, was also given.

Good luck!