Continuum Mechanics — Exam (Monday 31/05/2010)

Note: Be sure to write your name on each answer sheet. Hand in the solutions to the theory and the exercise exam separately. The exam runs from 14h until 18h.

T1. Show that the compatibility conditions of St-Venant are necessary and sufficient for a symmetric tensor E to represent a small-strain tensor.

T2. Let $f(z)$ be a complex potential describing a certain two-dimensional fluid flow around a disc with radius R.

(a) Derive the Blasius formula for the total force on the disc of radius R in the flow described by $f(z)$.

(b) Use this result to show that the total force on a cylinder of unit radius in a uniform flow with velocity U vanishes. You may use the expression $f(z) = U(z + 1/z)$ for the complex potential representing uniform flow around the unit disc.

O1. Consider the following tensor

$$\mathsf{E} = \begin{pmatrix} 3x^2 \sin \pi y & \frac{A\pi}{2} x^3 \cos \pi y + x & 0 \\ \frac{A\pi}{2} x^3 \cos \pi y + x & 0 & 0 \\ 0 & 0 & z \end{pmatrix},$$

where A is a constant.

(a) Determine A such that E represents a small-strain tensor.

(b) For this value of A, determine the deformation $\mathbf{u} = (u_x, u_y, u_z)$ if you know that points on the y-axis experience no deformation in the x-direction, and that the point with coordinates $(1, 1, 0)$ remains fixed. You may also assume that u_z depends only on z: $u_z = u_z(z)$.

(c) Sketch the deformation of a square in the xy-plane with corners at the points $A(1, 1)$, $B(-1, 1)$, $C(-1, -1)$, $D(1, -1)$.

(d) Under the assumption that the material satisfies Hooke's law with Lamé coefficients $\lambda = 1$ and $\mu = \frac{1}{2}$, write down the stress tensor and determine the force necessary to hold the medium in equilibrium (defined as $\mathbf{v} = 0$).

Q2. Consider the 2D potential flow with complex potential

$$f(z) = -\frac{K}{2\pi z},$$

where K is a real constant.

(a) Find the velocity potential ϕ, the stream function ψ and the complex velocity. Are there any stagnation points?

(b) Determine a parametric representation for the equipotential lines and the streamlines. Make a brief sketch of both sets of curves.

(c) Show explicitly that whenever the equipotential lines and the streamlines intersect, they do so at right angles.

(d) Discuss briefly the case where K is allowed to be a complex constant.