Exam Game theory: open book exam

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UGent, 08h30-11h45

QUESTION 1

Consider a worker whose ability is high (H) or low (L). He knows this and can invest in a degree D or not N. The cost of a degree is 2 if his ability is high and 5 if his ability is low. The employer has the choice to recruit the worker as manager (M) or as blue collar worker (B). The worker gets payed 10 if he's manager and 6 as a BCW. The payoff for the company is 10 if the worker's a high ability manager, 5 if he's a high ability BCW, 3 if he's a low ability BCW and 0 if he's a low ability manager.

- a (3pt) Both players know the worker's ability. Find the PSN, normal form, extended form and subgame perfect equilibria.
- b Incomplete information. The worker knows his ability, but the employer doesn't. The probability of H is $\frac{1}{4}$.
  1. (2pt) Give the extended form of the game.
  2. (2pt) Give normal form of the game and find the NE. Use the following notation.
     - Worker's strategy space: $xy \in S_W = \{NN, ND, DN, DD\}$ where x is what he does if H, and y is what he does if L.
     - Employer's strategy space: $vw \in S_E = \{MM, MB, BM, BB\}$ where v denotes what he does if the worker chooses N, and w denotes what he does if the worker chooses D.
- c (3pt) Show for the following strategy pairs under which conditions they are a PBNE in the game from (b). (NN, BB) and (DN, BM).

QUESTION 2 Two bidders, complete information. Let v_i be the value for $i = 1, 2$. Assume $v_1 > v_2$.

- a (2pt) Write down the payoffs of each player in function of their strategies.
- b (4pt) Derive the best response of both players.

- c (4pt) Make pictures in the $b_1 \times b_2$ plane, one of the first bidder's best response, one of the second bidder's best response and one of the Nash equilibria in pure strategies.

QUESTION 3 Consider a pairwise bilateral game. Define $\mathbf{x}(t) = (x_1(t), x_2(t))$ is the vector of population frequencies playing strategies S_1 and S_2 at time t, while $\pi(S_1, \mathbf{x}(t))$ and $\pi(S_2, \mathbf{x}(t))$ denote the payoffs of strategies S_1 and S_2 respectively.

- a (2pt) Lemma 1. The RD can be written as $\dot{x}(t) = x_1(t)[1-x_1(t)][\pi(S_1, \mathbf{x}(t)) - \pi(S_2, \mathbf{x}(t))$.
- b (8pt) Theorem. For any two-strategy bilateral game, a strategy is an ESS if and only if the corresponding rest point of the RD is asymptotically stable.

Reconstructed by Nathan Steyaert and Jeffrey De Rycke. We accept donations but only in the form of Carapils and ice tea.