Biophysics Exam

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Dear students. Good luck with the exam. Sometimes extra points can be earned by making an animation or widgets. If you are not so familiar with that, do it in the end if you have time left.

1 The quorum sensing - nullclines

Competence is a bacterial cell state in which DNA replication and cell division stops, and cells become able to acquire DNA from their surroundings. It has been proposed that the absorbed genetic material can provide templates for DNA repair, be a source of nutrients, or may simply be incorporated into the cell's genome. Competence occurs in bacterial populations with high cell densities.

In this exercise, we will examine a simplified model for the induction of competence in the bacterium *Bacillus subtilis* via quorum sensing. In *B. subtilis*, the transcription factor ComK activates the expression of several genes involved in making the cell competent, while also activating its own transcription. The degradation rate of ComK depends on ComS, a small peptide which all cells produce at a constant rate. As the density of a cell culture increases, the concentration of ComS increases too. Consequently, ComK expression increases as well and the cells become competent.

Our model has two species, K (representing ComK) and S (representing ComS) and its differential equations are provided below:

$$K' = \frac{80K}{1+100K} - \frac{K}{1+K+S} \tag{1}$$

$$S' = g_s - \frac{S}{1+K+S} \tag{2}$$

Keep in mind that the model describes concentrations of a transcription factor and a peptide, so negative values of S and K do not make sense biologically.

(a) Plot the nullclines, first assuming that $g_s = 0.1$. Then vary $g_s \in]0, 1[$. Extra points if you can make am animation or a widget. What do you see in terms of equilibria of your system?

(b) Calculate the equilibria exactly for $g_s \in]0, 1[$. Compute the Jacobian and determine the type of equilibria you have. Plot the equilibria in a 3D plot where K and S are your XY-plane and g_s is your Z-plane. Write a conclusion what you see.

(c) Divide your KS-plane in different parts for a given g_s devided by your nullclines (start with $g_s = 0.1$) and plot the direction of the vectorfield of different testpoints in the different zones of your plane. Also plot some typical trajectories of the differential equation. Plot this for different values of g_s. Extra points if you can make an animation or a widget.

2 The Rossler system

The simplest three-dimensional non-linear system of differential equations is the Rossler model:

$$\begin{aligned} X' &= -Y - Z \\ Y' &= X + aY \\ Z' &= b + XZ - cZ \end{aligned} \tag{3}$$

Solve the following questions.

(a) Currently we have seen the Euler equation for integrating systems. However, the Runge-Kutta method is much more precise for integrating dynamical systems. Suppose we have the following differential equation:

$$\begin{aligned} X' &= f(X,Y) \\ Y' &= g(X,Y) \end{aligned} \tag{4}$$

Implement the following integration scheme in your equations:

$$\begin{aligned} X_{n+1} &= X_n + \frac{h}{6}(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4}) \\ Y_{n+1} &= Y_n + \frac{h}{6}(\ell_{n1} + 2\ell_{n2} + 2\ell_{n3} + \ell_{n4}) \end{aligned} \tag{5}$$

with

$$\begin{aligned} k_{n1} &= f(X_n, Y_n) \\ \ell_{n1} &= g(X_n, Y_n) \\ k_{n2} &= f(X_n + \frac{h}{2}k_{n1}, Y_n + \frac{h}{2}\ell_{n1}) \\ \ell_{n2} &= g(X_n + \frac{h}{2}k_{n1}, Y_n + \frac{h}{2}\ell_{n1}) \\ k_{n3} &= f(X_n + \frac{h}{2}k_{n2}, Y_n + \frac{h}{2}\ell_{n2}) \\ \ell_{n3} &= g(X_n + \frac{h}{2}k_{n2}, Y_n + \frac{h}{2}\ell_{n2}) \\ k_{n4} &= f(X_n + hk_{n3}, Y_n + h\ell_{n3}) \\ \ell_{n4} &= g(X_n + hk_{n3}, Y_n + h\ell_{n3}) \end{aligned} \tag{6}$$

(b) Compare the Euler method with the Runge-Kutta method. Take the time step (h in the equation above) equal to 0.1 and check which time step you should take for the Euler equation to get the same precision. What do you notice if you also take 0.1 as time step for the Euler equation?

(c) Show the trajectory to chaos (set $a = b = 0.1$ and vary c) by making plots of the behavior in 3D. What do you see?

(d) Show that there is in fact chaos and no randomness in this system (you can chose you own method)

(e) Make a bifurcation diagram by plotting the maxima of X in function of c.