

# Theory exam Cosmology & Galaxy Formation, January 6, 2025

When asked to “explain” something, you must provide a physicist’s answer that consists of text, math, and (if necessary) a drawing or diagram. A qualitative description without any math to back it up does not suffice. Nor does a collection of mathematical equations without the necessary text to explain its meaning.

If a qualitative explanation suffices, this is explicitly mentioned.

- **question 1**

Explain the concept of **cosmological redshift** fully and thoroughly. What is it? Describe its physics using the theory developed in the syllabus. Provide two different explanations. Why is more than one explanation possible?

- **question 2**

Derive the **Hubble-Lemaître law**. Which cosmological parameters feature in this law? How can astronomers and cosmologists put this law to the observational test, and determine its parameters?

- **question 3**

Fully and thoroughly explain the **Gunn-Peterson effect**, both observationally and theoretically. What is it? Why is it important to cosmology? Provide a quantitative description.

- **question 4**

This question addresses structure formation in baryons after last scattering using linear perturbation theory.

1. Explain qualitatively why no baryon fluctuations exist, except on very large scales.
2. Explain qualitatively why dark-matter fluctuations do exist.
3. Explain quantitatively how baryon fluctuations evolve in a background of dark-matter fluctuations.

Hints:

$$0 = \frac{\partial \rho}{\partial t} + 3H\rho + \frac{\partial}{\partial x^i} (\rho u^i), \quad (1)$$

$$0 = \frac{\partial}{\partial t} (\rho u^i) + 5H\rho u^i + \frac{\rho}{R^2} \frac{\partial \phi}{\partial x^i}. \quad (2)$$

- **question 5**

List the main problems of classical cosmology. Describe **cosmic inflation**. Under what circumstances does it occur? Can we expect those circumstances to arise, and why? What effects does inflation produce? How does it solve the problems you listed? For the latter, you don’t need to consider problems that have to do with non-homogeneities.

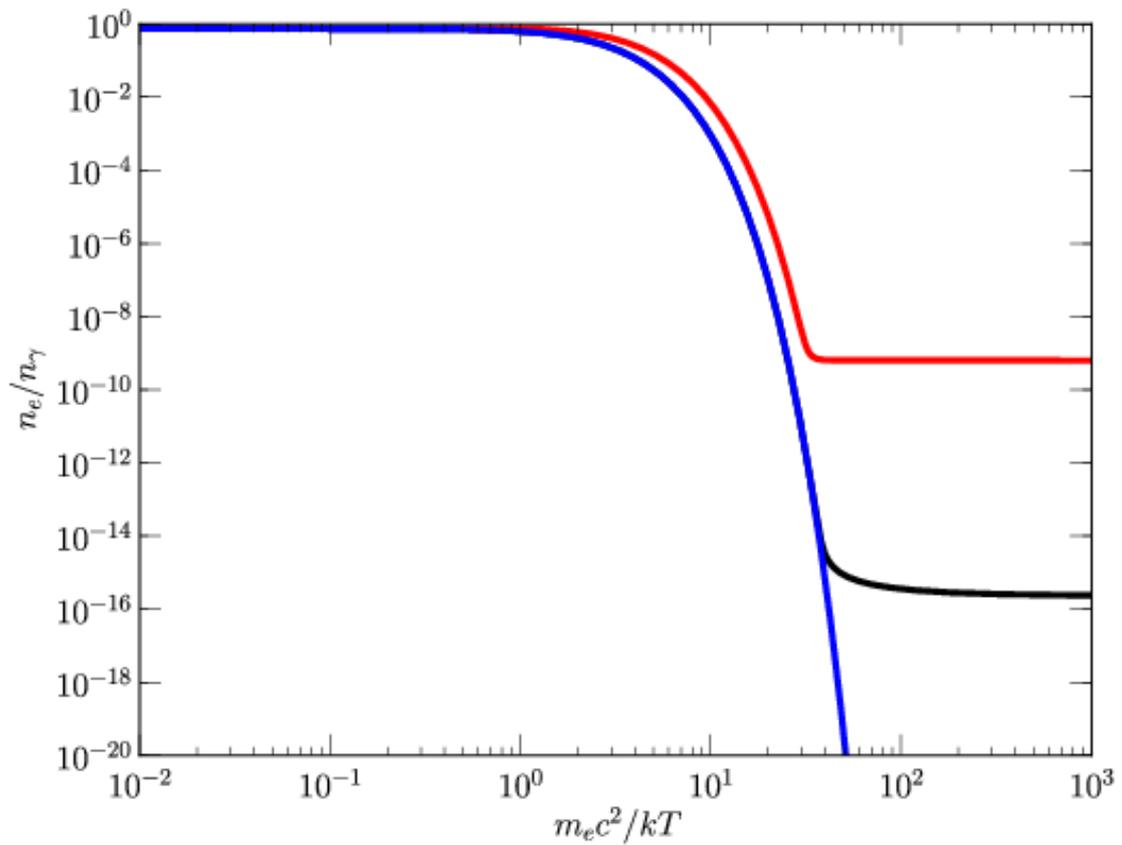
Hints:

$$\partial_\mu (\eta^{\mu\nu} \partial_\nu \phi) + \frac{\partial V}{\partial \phi} = 0, \quad (3)$$

$$V(\phi) = \frac{m^2 c^2}{2\hbar^2} \phi^2, \quad (4)$$

$$\rho c^2 = \frac{1}{2c^2} \dot{\phi}^2 + V(\phi), \quad (5)$$

$$p = \frac{1}{2c^2} \dot{\phi}^2 - V(\phi). \quad (6)$$



• **question 6 (oral)**

Discuss this diagram. What quantity is plotted? What is the difference between the different curves? Why do they behave differently? Base your answer on the theory we developed in the syllabus. Show that you understand the underlying physics. You don't need to provide calculations.

## Exercise exam Cosmology & Galaxy Formation, January 6, 2025

We investigate the evolution of a spatially localized **spherically symmetric overdensity** sitting inside an **Einstein-de Sitter universe**. The whole universe is matter dominated. We use **comoving** spherical coordinates  $(r, \theta, \phi)$ , anchored to the centre of the overdensity. The universe is isotropic around this centre, but it is not homogeneous. We assume that the density declines with increasing radius  $r$ .

Using these comoving coordinates, the  $ds^2$  can be written as

$$ds^2 = c^2 dt^2 - X(r, t)^2 dr^2 - R(r, t)^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (7)$$

The density depends both on time  $t$  and radius  $r$ , as  $\rho(r, t)$ .

In the remainder, partial  $t$ -derivatives are denoted by “ $\dot{\phantom{x}}$ ”, and partial  $r$ -derivatives by “ $\prime$ ”.

1. the (01)-component of the field equations reads as

$$\frac{\partial}{\partial t} \left( \frac{R'}{X} \right) = 0. \quad (8)$$

- Explain how this justifies the relation

$$X(r, t) = \frac{R'(r, t)}{\sqrt{1 - k(r)}}, \quad (9)$$

which immediately defines the function  $k(r)$ .

- Compare this with the Robertson-Walker metric of a homogeneous universe. What form does the function  $k(r)$  take there? How do you physically interpret the function  $k(r)$  here? Since the universe becomes Einstein-de Sitter far outside the overdensity, what do you then know about  $k(r = \infty)$ ? Given that we are dealing with an *overdensity*, what do you expect regarding the sign of the function  $k(r)$ ?
- What form does the scale function  $R(r, t)$  take in a Robertson-Walker universe?

2. The (00) and (11) components of the field equations now become

$$\frac{kc^2 + \dot{R}^2}{R^2} + \frac{k'c^2 + 2\dot{R}\dot{R}'}{RR'} = 8\pi G\rho \quad (10)$$

$$kc^2 + \dot{R}^2 + 2R\ddot{R} = 0. \quad (11)$$

- In total, we have the (00), (11), and (01) components of the field equations to work with. What is the physical reason we have one equation more than in the Robertson-Walker case?
- Show that eqn. (11) leads to the conclusion that

$$R\dot{R}^2 + kc^2R = F(r), \quad (12)$$

for some  $r$ -dependent function  $F$ .

- Show that eqn. (10) can be written as

$$8\pi G\rho(r, t) = \frac{F'(r)}{R(r, t)^2 R'(r, t)}, \quad (13)$$

and that this leads to the relation

$$F(r) = 8\pi G \int_0^r \rho(x, t) R(x, t)^2 R'(x, t) dx. \quad (14)$$

Explicitly compute  $F(r)$  for a Robertson-Walker universe. Give a simple, intuitively clear physical interpretation for  $F(r)$  based on this result.

Based on this interpretation, what do you expect the sign of  $R'$  to be? What does that tell you about the expansion rate of the universe as a function of radius  $r$ ?

3. As a boundary condition, suppose we know the scale function  $R(r, t)$  as a function of  $r$  at a given time  $t_0$ . In other words, we know  $R_0(r) = R(r, t_0)$ .

We define a ‘‘Hubble parameter’’ as

$$H(r, t) = \frac{\dot{R}(r, t)}{R(r, t)}, \quad (15)$$

and we take  $H_0(r) = H(r, t_0)$  as a known boundary condition. We define a new function  $\Omega_0(r)$  as

$$F(r) = H_0(r)^2 \Omega_0(r) R_0(r)^3. \quad (16)$$

Derive a relation that links  $k(r)$  to  $\Omega_0(r)$  (and a few other physical functions). To what important equation from the syllabus is it formally identical? Use this similarity to provide a physical interpretation for this equation.

4. Given that the density declines outwardly, and joins that of an Einstein-de Sitter universe outside of the overdensity, give a physical expectation for the behavior of  $\Omega_0(r)$ .
5. Use these results to rewrite eqn. (12) as a partial differential equation for  $R(r, t)$  involving only  $\Omega_0(r)$ ,  $H_0(r)$ , and  $R_0(r)$  as other quantities. Solve this Friedman-Lemaître-like equation (if you recognize its form, you can immediately lift the solution from the syllabus!).
6. Based on all former results, describe in words how the scale factor  $R(r, t)$  behaves as a function of time  $t$  at different radii  $r$ . Use this to describe how the spherical overdensity evolves with time.