Hoofdstuk 2: Eenvoudige Wiskundige Uitdrukkingen:

Oefening 1:

    (a) ((succ x) + (y * z))
       succ x + y * z

    (b) ((succ (x + y)) * z)
       succ (x + y) * z

Oefening 2:

    (a) z * succ (x+y) + x + y
       (((z * (succ (x + y))) + x) + y)

    (b) z + succ x * y
       (z + ((succ x) * y))

Oefening 4:

    (a) a + y

    (b) a * succ x + y

    (c) x * (z + a) + (z + a)

    (d) x * (z * a) + z * a

    (e) a + z

    (f) a * succ x + (a + (a + x))

    (g) (x + x * y) * (x * y) + x * y

    (h) a + z

    (i) a * succ x + (a + x)

    (j) (x + y) * (x * y) + x * y

Oefening 5:

    (a) uitdrukking

    (b) uitdrukking, formule

    (c) /

    (d) /

(e) /

(f) uitdrukking, formule

Oefening 6:

(a) ? = b * (x + a) + y
d = x
d' = x + a
e = (b * v + y)

(b) ? = x + b * z
d = a * y + 1
d' = b
e = (x + v * z)

(c) ? = a * y + a * x + 1
d = x+ 1
d' = y
e = (a * v + a * x + 1)

(d) d = x
d' = y
? = y + y
e = (v + v)
? = x + y
e = x + v
? = y + x
e = (v + x)

(e) d = b
d' = y + 1
? = (y + 1) * x + (y + 1) * y
e = (v * x + v * y)
? = b * x + (y + 1) * y
e = (b * x + v * y)
? = (y + 1) * x + b * y
e = (v * x + b * y)

Oefening 7:

(a) e = (v + y)
e |v := d'| = a + b + y

(b) e = x + y + x
e |v := d'| = x + y + x

(c) e = x * v
e |v := d'| = x * (y + x)

(d) e = (x + y) * v
     e |v := d'| = (x + y) * (x * y)

(e) e = (v * a * v)
     e |v := d'| = (y + x) * a * (y + x)

Oefening 8:

(a) e = (x + y) * v
     d = d' = x + y = y + x

(b) e = (v * v)
     d = d' = x + y = y + x

(c) e = (v + x)
     d = d' = x + y + a = x + y * a

(d) e = (v * y * x)
     d = d' =   x = y + a

(e) e = (v * x)
     d = d' =   x * y = y * x

Oefening 9:

$$\frac{e = e''}{d\ [v := e] = d\ [v := e'']}$$

Oefening 11:

- (x * (y + b) = x * y + x * b)|x := a|
  a * (y + b) = a * y + a * b

- (x * (y + b) = x * y + x * b)|y := a|
  x * (a + b) = x * a + x * b

- (x * (y + b) = x * y + x * b)|x, y := z * a, a|
  z * a * (a + b) = z * a * a + z * a * b

Oefening 12:

(a) x + a + y > b

(b) x + y = x

(c) y = x + (x + y)

Oefening 13:

(a) Principe van Leibniz

(b) Symmetrie van gelijkheid

(c) Instantiatie

(d) Symmetrie van gelijkheid

(e) Instantiatie

(f) Symmetrie van gelijkheid

(g) Principe van Leibniz

(h) Instantiatie

(i) Principe van Leibniz

(j) Symmetrie van gelijkheid

(k) Instantiatie

(l) Principe van Leibniz

(m) Principe van Leibniz

Oefening 14:

(a) Instantiatie

(b) Instantiatie + Principe van Leibniz

(c) Instantiatie

(d) Symmetrie van gelijkheid + Principe van Leibniz

(e) Instantiatie + Principe van Leibniz

(f) Instantiatie
Principe van Leibniz
Symmetrie van gelijkheid

Oefening 15:

(a) $z + a * x = \langle a * x = a \rangle z + a$
$= \langle a * y = a \rangle z + a * y$

(b) succ $a = \langle \text{succ } x = b + x \rangle b + a$
$= \langle x + a = x \rangle b$

(c) succ a + a * x = <succ x = b + x> b + x + a * x
              = <x + a = x> b + a * x
              = <z + a * x = z + a * y> b + a * y

Oefening 16:

(a) succ x   = <succ x = x + a> x + a

(b) succ a   = <succ x = x + a> a + a
              = <a + a = b> b

(c) a * b   = <x * y = y * x> b * a
              = <b * x = x + x> a + a
              = <a + a = b> b

(d) succ (x + y)   = <succ x = x + a> (x + y) +a
              = <(x + y) + z = x + (y + z)> x + (y + a)
              = <succ x = x + a> x + succ y

(e) succ (x + y)   = <succ x = x + a> (x + y) +a
              = <x + y = y + x> (y + x) + a
              = <(x + y) + z = x + (y + z)> y + (x + a)
              = <succ x = x + a> y + succ x
              = <x + y = y + x> succ x + y

Oefening 17:

(a) b = <b = a + a> a + a
       = <succ x = x + a> succ a

(b) b = <b = succ a> succ a
       = <succ x = x + a> a + a

(c) succ (succ x) = <succ x = x + a> succ (x + a)
              = <succ x = x + a> (x + a) + a
              = <(x + y) + z = x + (y + z)> x + (a + a)
              = <succ x = x + a> x + succ a
              = <b = succ a> x + b

(d) b * x = <b = succ a> (succ a) * x
              = <succ x = x + a> (a + a) * x
              = <(x + y) * z = x * z + y * z> a * x + a * x
              = <a * x = x> x + x

(e) b * x = <b = succ (succ a)> (succ (succ a)) * x
              = <succ x = x + a> succ (a + a) * x
              = <succ x = x + a> ((a + a) + a) * x
              = <(x + y) * z = x * z + y * z> (a + a) * x + a * x
              = <(x + y) * z = x * z + y * z> a * x + a * x + a * x
              = <a * x = x> x + x + x

(f) b * x * z   = <b = succ a, z = y> (succ a) * x * y
                = <succ x = x + a> (a + a) * x * y
                = <(x + y) * z = x * z + y * z> (a * x + a * x) * y
                = <a * x = x> (x + x) * y

Oefening 18:

a   = <x * b = x> a * b
    = <a * x = x> b