

## Hoofdstuk 4: Propositierekenen:

### Stelling 1:

$$\begin{aligned}x \equiv x \equiv y \equiv y &\equiv \langle \text{ax2 } (x \equiv y \equiv y) \equiv x \rangle x \equiv x \\ &\equiv \langle \text{ax2 } x \equiv (y \equiv y \equiv x) \rangle x \equiv y \equiv y \equiv x\end{aligned}$$

### Stelling 2:

$$\begin{aligned}1 &\equiv \langle \text{ax3, } 1 \equiv (y \equiv y) \text{ met } y := 1 \rangle 1 \equiv 1 \\ &\equiv \langle \text{ax3, } 1 \equiv (y \equiv y) \rangle 1 \equiv y \equiv y\end{aligned}$$

### Stelling 3:

$$\begin{aligned}x \equiv x &\equiv \langle \text{st1, } (x \equiv x) \equiv (y \equiv y) \rangle y \equiv y \\ &\equiv \langle \text{ax3, } 1 \equiv (y \equiv y) \rangle 1\end{aligned}$$

### Stelling 4:

$$\begin{aligned}y \equiv 1 \equiv y &\equiv \langle \text{ax3, } (1 \equiv y) \equiv y \rangle y \equiv y \\ &\equiv \langle \text{ax3, } 1 \equiv (y \equiv y) \rangle 1\end{aligned}$$

### Stelling 5:

$$\begin{aligned}\neg x \equiv y \equiv x \equiv \neg y &\equiv \langle \text{ax5, } \neg (x \equiv y) \equiv \neg x \equiv y \rangle \neg (x \equiv y) \equiv x \equiv \neg y \\ &\equiv \langle \text{ax2, } x \equiv y \equiv y \equiv x \text{ met } x, y := x, \neg y \rangle \neg (x \equiv y) \equiv \neg y \equiv x \\ &\equiv \langle \text{ax5, } \neg (x \equiv y) \equiv \neg x \equiv y \text{ met } x, y := y, x \rangle \neg (x \equiv y) \equiv \neg (y \equiv x) \\ &= \langle \text{ax2, } x \equiv y \equiv y \equiv x \text{ met } x, y := y, x \rangle \neg (x \equiv y) \equiv \neg (x \equiv y) \\ &= \langle \text{ax3, } 1 \equiv y \equiv y \text{ met } y := \neg (x \equiv y) \rangle 1\end{aligned}$$

$$\begin{aligned}\neg x \equiv y &\equiv \langle \text{ax5, } \neg (x \equiv y) \equiv \neg x \equiv y \rangle \neg (x \equiv y) \\ &\equiv \langle \text{ax2, } x \equiv y \equiv y \equiv x \rangle \neg (y \equiv x) \\ &= \langle \text{ax5, } \neg (x \equiv y) \equiv \neg x \equiv y \text{ met } x, y := y, x \rangle \neg y \equiv x \\ &= \langle \text{ax2, } x \equiv y \equiv y \equiv x \text{ met } x, y := \neg y, x \rangle x \equiv \neg y\end{aligned}$$

### Stelling 6:

$$\neg \neg x \equiv x \quad \equiv \langle \text{st5, met } x, y := \neg x, x \rangle \neg x \equiv \neg x$$

### Stelling 7:

$$\begin{aligned}\neg 0 &\equiv \langle \text{ax4} \rangle \neg \neg 1 \\ &\equiv \langle \text{st6} \rangle 1\end{aligned}$$

### Stelling 8:

$$\begin{aligned}(x \neq y) &\equiv \langle \text{ax6} \rangle \neg (x \equiv y) \\ &\equiv \langle \text{ax5} \rangle \neg x \equiv y\end{aligned}$$

### Stelling 9:

$$\begin{aligned}\neg x \equiv x &\equiv \langle \text{ax5} \rangle \neg (x \equiv x) \\ &\equiv \langle \text{ax3} \rangle \neg 1 \\ &\equiv \langle \text{ax4} \rangle 0\end{aligned}$$

### Stelling 10:

$$\begin{aligned}(x \not\equiv y) &\equiv \langle \text{ax6} \rangle \neg (x \equiv y) \\ &\equiv \langle \text{ax2} \rangle \neg (y \equiv x) \\ &\equiv \langle \text{ax6} \rangle (y \not\equiv x)\end{aligned}$$

### Stelling 11:

$$\begin{aligned}((x \not\equiv y) \not\equiv z) &\equiv \langle \text{st8} \rangle (\neg x \equiv y) \not\equiv z \\ &\equiv \langle \text{st8} \rangle \neg (\neg x \equiv y) \equiv z \\ &\equiv \langle \text{ax5} \rangle (\neg \neg x \equiv y) \equiv z \\ &\equiv \langle \text{st6} \rangle (x \equiv y) \equiv z \\ &\equiv \langle \text{ax1} \rangle x \equiv (y \equiv z) \\ &\equiv \langle \text{st6} \rangle x \equiv (\neg \neg y \equiv z) \\ &\equiv \langle \text{ax5} \rangle x \equiv \neg (\neg y \equiv z) \\ &\equiv \langle \text{ax2} \rangle \neg (\neg y \equiv z) \equiv x \\ &\equiv \langle \text{ax5} \rangle \neg ((\neg y \equiv z) \equiv x) \\ &\equiv \langle \text{ax2} \rangle \neg (x \equiv (\neg y \equiv z)) \\ &\equiv \langle \text{ax5} \rangle (\neg x \equiv (\neg y \equiv z)) \\ &\equiv \langle \text{st8} \rangle (x \not\equiv (\neg y \equiv z)) \\ &\equiv \langle \text{st8} \rangle (x \not\equiv (y \not\equiv z))\end{aligned}$$

### Stelling 12:

$$\begin{aligned}((x \not\equiv y) \equiv z) &\equiv \langle \text{ax6} \rangle (\neg (x \equiv y) \equiv z) \\ &\equiv \langle \text{ax5} \rangle \neg ((x \equiv y) \equiv z) \\ &\equiv \langle \text{ax1} \rangle \neg (x \equiv (y \equiv z)) \\ &\equiv \langle \text{ax6} \rangle (x \not\equiv (y \equiv z))\end{aligned}$$

### Stelling 13:

$$\begin{aligned}x \not\equiv y \equiv z &\equiv \langle \text{ax6} \rangle \neg (x \equiv (y \equiv z)) \\ &\equiv \langle \text{ax2} \rangle \neg ((y \equiv z) \equiv x) \\ &\equiv \langle \text{ax5} \rangle (\neg (y \equiv z) \equiv x) \\ &\equiv \langle \text{ax2} \rangle (x \equiv \neg (y \equiv z)) \\ &\equiv \langle \text{ax6} \rangle x \equiv y \not\equiv z\end{aligned}$$

### Stelling 14:

$$\begin{aligned}x \vee 1 &\equiv \langle \text{ax3} \rangle x \vee (x \equiv x) \\ &\equiv \langle \text{ax10} \rangle x \vee x \equiv x \vee x \\ &\equiv \langle \text{ax9} \rangle x \equiv x \\ &= \langle \text{ax3} \rangle 1\end{aligned}$$

### Stelling 15:

$$\begin{aligned}x \vee 0 &\equiv \langle \text{st9} \rangle x \vee (\neg x \equiv x) \\ &\equiv \langle \text{ax10} \rangle x \vee (\neg x) \equiv x \vee x \\ &\equiv \langle \text{ax11}, \text{ax9} \rangle 1 \equiv x \\ &\equiv \langle \text{ax3} \rangle x\end{aligned}$$

### Stelling 16:

$$\begin{aligned}(x \vee y) \vee (x \vee z) &\equiv \langle \text{ax8} \rangle x \vee (y \vee (x \vee z)) \\ &\equiv \langle \text{ax8} \rangle x \vee ((y \vee x) \vee z) \\ &\equiv \langle \text{ax7} \rangle x \vee ((x \vee y) \vee z) \\ &\equiv \langle \text{ax8} \rangle x \vee (x \vee (y \vee z)) \\ &\equiv \langle \text{ax8} \rangle (x \vee x) \vee (y \vee z) \\ &\equiv \langle \text{ax9} \rangle x \vee (y \vee z)\end{aligned}$$

### Stelling 17:

$$\begin{aligned}x \vee y \equiv x \vee \neg y &\equiv \langle \text{ax10} \rangle x \vee (y \equiv \neg y) \\ &\equiv \langle \text{ax2} \rangle x \vee (\neg y \equiv y) \\ &\equiv \langle \text{st9} \rangle x \vee 0 \\ &= \langle \text{st15} \rangle x\end{aligned}$$

### Stelling 18:

$$\begin{aligned}x \wedge y &\equiv \langle \text{ax12} \rangle x \equiv y \equiv x \vee y \\ &\equiv \langle \text{ax2} \rangle y \equiv x \equiv x \vee y \\ &\equiv \langle \text{ax7} \rangle y \equiv x \equiv y \vee x \\ &\equiv \langle \text{ax12} \rangle y \wedge x\end{aligned}$$

### Stelling 19:

$$\begin{aligned}(x \wedge y) \wedge z &\equiv \langle \text{ax12} \rangle (x \equiv y \equiv x \vee y) \wedge z \\ &\equiv \langle \text{ax12} \rangle (x \equiv y \equiv x \vee y) \equiv z \equiv (x \equiv y \equiv x \vee y) \vee z \\ &\equiv \langle \text{ax10} \rangle x \equiv y \equiv x \vee y \equiv z \equiv (x \equiv y) \vee z \equiv (x \vee y) \vee z \\ &\equiv \langle \text{ax10}, \text{ax8} \rangle x \equiv y \equiv x \vee y \equiv z \equiv x \vee z \equiv y \vee z \equiv (y \vee z) \vee x \\ &\equiv \langle \text{ax2}, \text{ax10} \rangle y \equiv z \equiv y \vee z \equiv x \equiv (y \equiv z) \vee x \equiv (y \vee z) \vee x \\ &\equiv \langle \text{ax10} \rangle (y \equiv z \equiv y \vee z) \equiv x \equiv (y \equiv z \equiv y \vee z) \vee x \\ &\equiv \langle \text{ax12} \rangle (y \equiv z \equiv y \vee z) \wedge x \\ &\equiv \langle \text{ax12} \rangle (y \wedge z) \wedge x \\ &\equiv \langle \text{st18} \rangle x \wedge (y \wedge z)\end{aligned}$$

### Stelling 20:

$$\begin{aligned}x \wedge x &\equiv \langle \text{ax12} \rangle x \equiv x \equiv x \vee x \\ &\equiv \langle \text{ax9} \rangle x \equiv x \equiv x \\ &\equiv \langle \text{ax3} \rangle 1 \equiv x \\ &\equiv \langle \text{ax3} \rangle x\end{aligned}$$

### Stelling 21:

$$\begin{aligned}
x \wedge 1 &\equiv \langle \text{ax12} \rangle x \equiv 1 \equiv x \vee 1 \\
&\equiv \langle \text{st14} \rangle x \equiv 1 \equiv 1 \\
&\equiv \langle \text{ax2} \rangle x
\end{aligned}$$

Stelling 22:

$$\begin{aligned}
x \wedge 0 &\equiv \langle \text{ax12} \rangle x \equiv 0 \equiv x \vee 0 \\
&\equiv \langle \text{st15} \rangle x \equiv 0 \equiv x \\
&\equiv \langle \text{st9} \rangle \neg x \equiv x \\
&\equiv \langle \text{st9} \rangle 0
\end{aligned}$$

Stelling 23:

$$\begin{aligned}
(x \wedge y) \wedge (x \wedge z) &\equiv \langle \text{st19} \rangle x \wedge (y \wedge (x \wedge z)) \\
&\equiv \langle \text{st18} \rangle x \wedge ((x \wedge z) \wedge y) \\
&\equiv \langle \text{st19} \rangle x \wedge (x \wedge (z \wedge y)) \\
&\equiv \langle \text{st19} \rangle (x \wedge x) \wedge (z \wedge y) \\
&\equiv \langle \text{st20} \rangle x \wedge (z \wedge y) \\
&\equiv \langle \text{st18} \rangle x \wedge (y \wedge z)
\end{aligned}$$

Stelling 24:

$$\begin{aligned}
x \wedge \neg x &\equiv \langle \text{st9} \rangle x \wedge (x \equiv 0) \\
&\equiv \langle \text{ax12} \rangle x \equiv (x \equiv 0) \equiv x \vee (x \equiv 0) \\
&\equiv \langle \text{ax10} \rangle x \equiv (x \equiv 0) \equiv x \vee x \equiv x \vee 0 \\
&\equiv \langle \text{st9}, \text{ax9}, \text{st15} \rangle x \equiv \neg x \equiv x \equiv x \\
&\equiv \langle \text{ax2} \rangle x \equiv \neg x \\
&\equiv \langle \text{ax2} \rangle \neg x \equiv x \\
&\equiv \langle \text{st9} \rangle 0
\end{aligned}$$

Stelling 25: (a):

$$\begin{aligned}
x \wedge (x \vee y) &\equiv \langle \text{ax12} \rangle x \equiv (x \vee y) \equiv x \vee (x \vee y) \\
&\equiv \langle \text{ax2}, \text{ax8} \rangle (x \vee y) \equiv x \equiv ((x \vee x) \vee y) \\
&\equiv \langle \text{ax9} \rangle (x \vee y) \equiv x \equiv (x \vee y) \\
&\equiv \langle \text{ax2} \rangle x \equiv (x \vee y) \equiv (x \vee y) \\
&\equiv \langle \text{ax2} \rangle x
\end{aligned}$$

Stelling 25: (b):

$$\begin{aligned}
x \vee (x \wedge y) &\equiv \langle \text{ax12} \rangle x \wedge (x \wedge y) \equiv x \equiv (x \wedge y) \\
&\equiv \langle \text{st19} \rangle ((x \wedge x) \wedge y) \equiv x \equiv (x \wedge y) \\
&\equiv \langle \text{st20} \rangle (x \wedge y) \equiv x \equiv (x \wedge y) \\
&\equiv \langle \text{ax2} \rangle x \equiv (x \wedge y) \equiv (x \wedge y) \\
&\equiv \langle \text{ax2} \rangle x
\end{aligned}$$

Stelling 26: (a):

$$x \wedge (\neg x \vee y) \equiv \langle \text{ax12} \rangle x \equiv (\neg x \vee y) \equiv x \vee (\neg x \vee y)$$

$$\begin{aligned}
&\equiv \langle \text{st16} \rangle x \equiv (\neg x \vee y) \equiv (x \vee \neg x) \vee (x \vee y) \\
&\equiv \langle \text{st9}, \text{ax11} \rangle x \equiv ((x \equiv 0) \vee y) \equiv 1 \vee (x \vee y) \\
&\equiv \langle \text{ax7}, \text{st16} \rangle x \equiv (y \vee (x \equiv 0)) \equiv (1 \vee x) \vee (1 \vee y) \\
&\equiv \langle \text{ax10}, \text{ax7} \rangle x \equiv (y \vee x) \equiv (y \vee 0) \equiv (x \vee 1) \vee (y \vee 1) \\
&\equiv \langle \text{st15}, \text{st14} \rangle x \equiv (y \vee x) \equiv y \equiv (1 \vee 1) \\
&\equiv \langle \text{ax7}, \text{st14} \rangle x \equiv (x \vee y) \equiv y \equiv 1 \\
&\equiv \langle \text{ax2}, \text{ax3} \rangle x \equiv (x \vee y) \equiv y \\
&\equiv \langle \text{ax2} \rangle x \equiv y \equiv (x \vee y) \\
&\equiv \langle \text{ax12} \rangle x \wedge y
\end{aligned}$$

Stelling 26: (b):

$$\begin{aligned}
x \vee (\neg x \wedge y) &\equiv \langle \text{ax12} \rangle x \vee (\neg x \equiv y \equiv (\neg x \vee y)) \\
&\equiv \langle \text{ax10} \rangle x \vee (\neg x \equiv y) \equiv x \vee (\neg x \vee y) \\
&\equiv \langle \text{ax8} \rangle x \vee (\neg x \equiv y) \equiv (x \vee \neg x) \vee y \\
&\equiv \langle \text{ax11} \rangle x \vee (\neg x \equiv y) \equiv 1 \vee y \\
&\equiv \langle \text{ax7} \rangle x \vee (\neg x \equiv y) \equiv y \vee 1 \\
&\equiv \langle \text{st14} \rangle x \vee (\neg x \equiv y) \equiv 1 \\
&\equiv \langle \text{ax10} \rangle x \vee \neg x \equiv x \vee y \equiv 1 \\
&\equiv \langle \text{ax2} \rangle x \vee y \equiv x \vee \neg x \equiv 1 \\
&\equiv \langle \text{ax11} \rangle x \vee y \equiv 1 \\
&\equiv \langle \text{ax3} \rangle x \vee y
\end{aligned}$$

Stelling 27:

$$\begin{aligned}
x \vee (y \wedge z) &\equiv \langle \text{ax12} \rangle x \vee (y \equiv z \equiv y \vee z) \\
&\equiv \langle \text{ax10} \rangle x \vee (y \equiv z) \equiv x \vee (y \vee z) \\
&\equiv \langle \text{ax10}, \text{st16} \rangle x \vee y \equiv x \vee z \equiv (x \vee y) \vee (x \vee z) \\
&\equiv \langle \text{ax12} \rangle (x \vee y) \wedge (x \vee z)
\end{aligned}$$

Stelling 28:

$$\begin{aligned}
(x \wedge y) \vee (x \wedge z) &\equiv \langle \text{st27} \rangle ((x \wedge y) \vee x) \wedge ((x \wedge y) \vee z) \\
&\equiv \langle \text{ax2} \rangle (x \vee (x \wedge y)) \wedge (z \vee (x \wedge y)) \\
&\equiv \langle \text{st25b} \rangle x \wedge (z \vee (x \wedge y)) \\
&\equiv \langle \text{st27} \rangle x \wedge ((z \vee x) \wedge (z \vee y)) \\
&\equiv \langle \text{ax2} \rangle x \wedge ((x \vee z) \wedge (z \vee y)) \\
&\equiv \langle \text{st25a} \rangle x \wedge (z \vee y)
\end{aligned}$$

Stelling 29: (a):

$$\begin{aligned}
\neg(x \wedge y) &\equiv \langle \text{ax12} \rangle \neg(x \equiv y \equiv x \vee y) \\
&\equiv \langle \text{ax5} \rangle \neg x \equiv y \equiv x \vee y \\
&\equiv \langle \text{ax7} \rangle \neg x \equiv y \equiv y \vee x \\
&\equiv \langle \text{st17} \rangle \neg x \equiv y \vee \neg x \\
&\equiv \langle \text{st6} \rangle \neg x \equiv \neg \neg y \vee \neg x \\
&\equiv \langle \text{ax7} \rangle \neg x \equiv \neg x \vee \neg \neg y \\
&\equiv \langle \text{st17} \rangle \neg x \vee \neg y
\end{aligned}$$

Stelling 29: (b):

$$\begin{aligned}\neg(x \vee y) &\equiv \langle \text{st17} \rangle \neg(x \vee \neg y \equiv x) \\ &\equiv \langle \text{ax5} \rangle (x \vee \neg y) \equiv \neg x \\ &\equiv \langle \text{st9} \rangle ((\neg x \equiv 0) \vee \neg y) \equiv \neg x \\ &\equiv \langle \text{ax7, ax10} \rangle (\neg y \vee \neg x \equiv \neg y \vee 0) \equiv \neg x \\ &\equiv \langle \text{ax7, st15} \rangle \neg x \vee \neg y \equiv \neg y \equiv \neg x \\ &\equiv \langle \text{ax2} \rangle \neg y \equiv \neg x \equiv \neg x \vee \neg y \\ &\equiv \langle \text{ax2} \rangle \neg x \equiv \neg y \equiv \neg x \vee \neg y \\ &\equiv \langle \text{ax12} \rangle \neg x \wedge \neg y\end{aligned}$$

Stelling 30:

$$\begin{aligned}x \wedge y \equiv x \wedge \neg y &\equiv \langle \text{st6} \rangle \neg \neg x \wedge y \equiv \neg \neg x \wedge \neg y \\ &\equiv \langle \text{st6} \rangle \neg \neg x \wedge \neg \neg y \equiv \neg \neg x \wedge \neg y \\ &\equiv \langle \text{st29b} \rangle \neg(\neg x \vee \neg y) \equiv \neg(\neg x \vee y) \\ &\equiv \langle \text{st5} \rangle \neg x \vee \neg y \equiv \neg x \vee y \\ &\equiv \langle \text{st17} \rangle \neg x\end{aligned}$$

Stelling 31:

$$\begin{aligned}x \wedge (y \equiv z) \equiv x \wedge y \equiv x \wedge z &\equiv \langle \text{ax12} \rangle x \equiv y \equiv z \equiv x \vee (y \equiv z) \equiv x \equiv y \equiv x \vee y \equiv x \equiv z \equiv x \vee z \\ &\equiv \langle \text{ax10} \rangle x \equiv y \equiv z \equiv x \vee y \equiv x \vee z \equiv x \equiv y \equiv x \vee y \equiv x \equiv z \equiv x \vee z \\ &\equiv \langle \text{ax2} \rangle x \equiv y \equiv z \equiv x \equiv z \equiv x \equiv y \equiv x \vee y \equiv x \vee z \equiv x \vee z \\ &\equiv \langle \text{ax3} \rangle x \equiv y \equiv z \equiv x \equiv z \equiv x \equiv y \equiv x \vee y \equiv x \vee z \equiv 1 \\ &\equiv \langle \text{ax3} \rangle x \equiv y \equiv z \equiv x \equiv z \equiv x \equiv y \equiv 1 \equiv 1 \\ &\equiv \langle \text{ax2} \rangle y \equiv y \equiv z \equiv x \equiv z \equiv x \equiv x \equiv 1 \equiv 1 \\ &\equiv \langle \text{ax2} \rangle y \equiv x \equiv z \equiv y \equiv z \equiv x \equiv x \equiv 1 \equiv 1 \\ &\equiv \langle \text{ax2} \rangle y \equiv x \equiv z \equiv y \equiv x \equiv z \equiv x \equiv 1 \equiv 1 \\ &\equiv \langle \text{ax3} \rangle 1 \equiv x \equiv 1 \equiv 1 \\ &\equiv \langle \text{ax2} \rangle x \equiv 1 \equiv 1 \equiv 1 \\ &\equiv \langle \text{ax3} \rangle x \equiv 1 \\ &\equiv \langle \text{ax3} \rangle x\end{aligned}$$

Stelling 32:

$$\begin{aligned}x \wedge (y \equiv x) &\equiv \langle \text{ax12} \rangle x \equiv (y \equiv x) \equiv x \vee (y \equiv x) \\ &\equiv \langle \text{ax1, ax10} \rangle x \equiv y \equiv x \equiv x \vee y \equiv x \vee x \\ &\equiv \langle \text{ax9, ax2} \rangle x \equiv x \equiv y \equiv x \vee y \equiv x \\ &\equiv \langle \text{ax3} \rangle 1 \equiv y \equiv x \vee y \equiv x \\ &\equiv \langle \text{ax3} \rangle y \equiv x \vee y \equiv x \\ &\equiv \langle \text{ax2} \rangle x \equiv y \equiv x \vee y \\ &\equiv \langle \text{ax12} \rangle x \wedge y\end{aligned}$$

Stelling 33:

$$\begin{aligned}(x \equiv y) \wedge (z \equiv x) &\equiv \langle \text{st31} \rangle (x \equiv y) \wedge z \equiv (x \equiv y) \wedge x \equiv (x \equiv y) \\ &\equiv \langle \text{st18} \rangle z \wedge (x \equiv y) \equiv x \wedge (x \equiv y) \equiv (x \equiv y) \\ &\equiv \langle \text{st31} \rangle z \wedge x \equiv z \wedge y \equiv z \equiv x \wedge x \equiv x \wedge y \equiv x \equiv x \equiv y \\ &\equiv \langle \text{st20} \rangle z \wedge x \equiv z \wedge y \equiv z \equiv x \equiv x \wedge y \equiv x \equiv x \equiv y\end{aligned}$$

$$\begin{aligned}
&\equiv \langle \text{ax2} \rangle z \wedge x \equiv z \wedge y \equiv z \equiv x \wedge y \equiv x \equiv x \equiv x \equiv y \\
&\equiv \langle \text{ax3} \rangle z \wedge x \equiv z \wedge y \equiv z \equiv x \wedge y \equiv 1 \equiv x \equiv y \\
&\equiv \langle \text{ax3} \rangle z \wedge x \equiv z \wedge y \equiv z \equiv x \wedge y \equiv y \equiv y \equiv x \equiv y \\
&\equiv \langle \text{st20} \rangle z \wedge x \equiv z \wedge y \equiv z \equiv x \wedge y \equiv y \wedge y \equiv y \equiv x \equiv y \\
&\equiv \langle \text{st18} \rangle z \wedge x \equiv z \wedge y \equiv z \equiv y \wedge x \equiv y \wedge y \equiv y \equiv x \equiv y \\
&\equiv \langle \text{st31} \rangle z \wedge (x \equiv y) \equiv y \wedge (x \equiv y) \equiv x \equiv y \\
&\equiv \langle \text{st18} \rangle (x \equiv y) \wedge z \equiv (x \equiv y) \wedge y \equiv (x \equiv y) \\
&\equiv \langle \text{st31} \rangle (x \equiv y) \wedge (z \equiv y)
\end{aligned}$$

Stelling 34:

$$\begin{aligned}
(x \wedge y) \vee (\neg x \wedge \neg y) &\equiv \langle \text{ax12} \rangle (x \equiv y \equiv x \vee y) \vee (\neg x \wedge \neg y) \\
&\equiv \langle \text{st29b} \rangle (x \equiv y \equiv x \vee y) \vee (\neg (x \vee y)) \\
&\equiv \langle \text{st18} \rangle (\neg (x \vee y)) \vee (x \equiv y \equiv x \vee y) \\
&\equiv \langle \text{ax10} \rangle (\neg (x \vee y)) \vee (x \equiv y) \equiv (\neg (x \vee y)) \vee (x \vee y) \\
&\equiv \langle \text{ax11} \rangle (\neg (x \vee y)) \vee (x \equiv y) \equiv 1 \\
&\equiv \langle \text{st17} \rangle (x \vee y) \vee (x \equiv y) \equiv (x \equiv y) \equiv 1 \\
&\equiv \langle \text{ax10} \rangle (x \vee y) \vee x \equiv (x \vee y) \vee y \equiv (x \equiv y) \equiv 1 \\
&\equiv \langle \text{ax7, ax8} \rangle y \vee (x \vee x) \equiv x \vee (y \vee y) \equiv (x \equiv y) \equiv 1 \\
&\equiv \langle \text{ax9} \rangle y \vee x \equiv x \vee y \equiv (x \equiv y) \equiv 1 \\
&\equiv \langle \text{ax7} \rangle y \vee x \equiv y \vee x \equiv (x \equiv y) \equiv 1 \\
&\equiv \langle \text{ax2} \rangle (x \equiv y) \equiv 1 \\
&\equiv \langle \text{st4} \rangle x \equiv y
\end{aligned}$$

Stelling 35:

$$\begin{aligned}
(\neg x \wedge y) \vee (x \wedge \neg y) &\equiv \langle \text{st6} \rangle (\neg x \wedge y) \vee (\neg \neg x \wedge \neg y) \\
&\equiv \langle \text{st34} \rangle \neg x \equiv y \\
&\equiv \langle \text{st8} \rangle x \not\equiv y
\end{aligned}$$

Stelling 36:

$$\begin{aligned}
x \Rightarrow y &\equiv \langle \text{ax13} \rangle x \vee y \equiv y \\
&\equiv \langle \text{ax7} \rangle y \vee x \equiv y \\
&\equiv \langle \text{st17} \rangle y \vee \neg x \equiv y \equiv y \\
&\equiv \langle \text{ax2} \rangle y \vee \neg x \\
&\equiv \langle \text{ax7} \rangle \neg x \vee y
\end{aligned}$$

Stelling 37:

$$\begin{aligned}
x \Rightarrow y &\equiv \langle \text{ax13} \rangle x \vee y \equiv y \\
&\equiv \langle \text{ax12} \rangle x \wedge y \equiv x
\end{aligned}$$

Stelling 38:

$$\begin{aligned}
x \Rightarrow y &\equiv \langle \text{ax13} \rangle x \vee y \equiv y \\
&\equiv \langle \text{st17} \rangle x \vee \neg y \equiv x \equiv y \\
&\equiv \langle \text{ax7} \rangle \neg y \vee x \equiv x \equiv y \\
&\equiv \langle \text{st17} \rangle \neg y \vee \neg x \equiv \neg y \equiv x \equiv y \\
&\equiv \langle \text{st5} \rangle \neg y \vee \neg x \equiv \neg x
\end{aligned}$$

$$\equiv \langle \text{ax13} \rangle \neg y \Rightarrow \neg x$$

Stelling 39:

$$\begin{aligned} x \Rightarrow (y \equiv z) &\equiv \langle \text{st37} \rangle x \wedge (y \equiv z) \equiv x \\ &\equiv \langle \text{st31} \rangle x \wedge y \equiv x \wedge z \equiv x \equiv x \\ &\equiv \langle \text{ax2} \rangle x \wedge y \equiv x \wedge z \end{aligned}$$

Stelling 40:

$$\begin{aligned} x \Rightarrow (y \equiv z) &\equiv \langle \text{st37} \rangle x \wedge (y \equiv z) \equiv x \\ &\equiv \langle \text{st31} \rangle x \wedge y \equiv x \wedge z \equiv x \equiv x \\ &\equiv \langle \text{ax2} \rangle x \wedge y \equiv x \equiv x \wedge z \equiv x \\ &\equiv \langle \text{st37} \rangle x \Rightarrow y \equiv x \Rightarrow z \end{aligned}$$

Stelling 41:

$$\begin{aligned} (x \Rightarrow y) \Rightarrow (x \Rightarrow z) &\equiv \langle \text{ax13} \rangle (x \vee y \equiv y) \Rightarrow (x \vee z \equiv z) \\ &\equiv \langle \text{ax13} \rangle (x \vee y \equiv y) \vee (x \vee z \equiv z) \equiv (x \vee z \equiv z) \\ &\equiv \langle \text{ax10} \rangle (x \vee y \equiv y) \vee (x \vee z) \equiv (x \vee y \equiv y) \vee z \equiv (x \vee z \equiv z) \\ &\equiv \langle \text{ax10} \rangle (x \vee z) \vee (x \vee y) \equiv (x \vee z) \vee y \equiv z \vee (x \vee y) \equiv z \vee y \equiv x \vee z \equiv z \\ &\equiv \langle \text{ax9} \rangle x \vee z \vee y \equiv x \vee z \vee y \equiv x \vee z \vee y \equiv z \vee y \equiv x \vee z \equiv z \\ &\equiv \langle \text{ax3} \rangle x \vee z \vee y \equiv z \vee y \equiv x \vee z \equiv z \\ &\equiv \langle \text{ax10} \rangle x \vee (y \vee z \equiv z) \equiv (y \vee z \equiv z) \\ &\equiv \langle \text{ax13} \rangle x \Rightarrow (y \vee z \equiv z) \\ &\equiv \langle \text{ax13} \rangle x \Rightarrow (y \Rightarrow z) \end{aligned}$$

Stelling 42:

$$\begin{aligned} (x \wedge y) \Rightarrow z &\equiv \langle \text{st30} \rangle \neg (x \wedge y) \vee z \\ &\equiv \langle \text{st29a} \rangle (\neg x \vee \neg y) \vee z \\ &\equiv \langle \text{ax8} \rangle \neg x \vee (\neg y \vee z) \\ &\equiv \langle \text{st30} \rangle x \Rightarrow (\neg y \vee z) \\ &\equiv \langle \text{st30} \rangle x \Rightarrow (y \Rightarrow z) \end{aligned}$$

Stelling 43:

$$\begin{aligned} x \wedge (x \Rightarrow y) &\equiv \langle \text{st36} \rangle x \wedge (\neg x \vee y) \\ &\equiv \langle \text{st26a} \rangle x \wedge y \end{aligned}$$

Stelling 44:

$$\begin{aligned} x \wedge (y \Rightarrow x) &\equiv \langle \text{st38} \rangle x \wedge (\neg x \Rightarrow \neg y) \\ &\equiv \langle \text{st36} \rangle x \wedge (\neg \neg x \vee \neg y) \\ &\equiv \langle \text{st6} \rangle x \wedge (x \vee \neg y) \\ &\equiv \langle \text{st25a} \rangle x \end{aligned}$$

Stelling 45:

$$x \vee (x \Rightarrow y) \equiv \langle \text{st36} \rangle x \vee (\neg x \vee y)$$



$$\begin{aligned}
&\equiv \langle \text{ax8} \rangle (x \vee \neg x) \vee y \\
&\equiv \langle \text{ax11} \rangle 1 \vee y \\
&\equiv \langle \text{ax7} \rangle y \vee 1 \\
&\equiv \langle \text{st14} \rangle 1
\end{aligned}$$

Stelling 46:

$$\begin{aligned}
x \vee (y \Rightarrow x) &\equiv \langle \text{st36} \rangle x \vee (\neg y \vee x) \\
&\equiv \langle \text{ax7} \rangle x \vee (x \vee \neg y) \\
&\equiv \langle \text{ax8} \rangle (x \vee x) \vee \neg y \\
&\equiv \langle \text{ax9} \rangle x \vee \neg y \\
&\equiv \langle \text{ax7} \rangle \neg y \vee x \\
&\equiv \langle \text{st36} \rangle y \Rightarrow x
\end{aligned}$$

Stelling 47:

$$\begin{aligned}
(x \vee y) \Rightarrow (x \wedge y) &\equiv \langle \text{st36} \rangle \neg (x \vee y) \vee (x \wedge y) \\
&\equiv \langle \text{st29b} \rangle (\neg x \wedge \neg y) \vee (x \wedge y) \\
&\equiv \langle \text{ax7} \rangle (x \wedge y) \vee (\neg x \wedge \neg y) \\
&\equiv \langle \text{st34} \rangle x \equiv y
\end{aligned}$$

Stelling 48:

$$\begin{aligned}
x \Rightarrow x &\equiv \langle \text{st36} \rangle \neg x \vee x \\
&\equiv \langle \text{ax7} \rangle x \vee \neg x \\
&\equiv \langle \text{ax11} \rangle 1
\end{aligned}$$

Stelling 49:

$$\begin{aligned}
x \Rightarrow 1 &\equiv \langle \text{st36} \rangle \neg x \vee 1 \\
&\equiv \langle \text{st17} \rangle x \vee 1 \equiv 1 \\
&\equiv \langle \text{st14} \rangle 1 \equiv 1 \\
&\equiv \langle \text{ax3} \rangle 1
\end{aligned}$$

Stelling 50:

$$\begin{aligned}
1 \Rightarrow x &\equiv \langle \text{st36} \rangle \neg 1 \vee x \\
&\equiv \langle \text{ax7} \rangle x \vee \neg 1 \\
&\equiv \langle \text{ax4} \rangle x \vee 0 \\
&\equiv \langle \text{st15} \rangle x
\end{aligned}$$

Stelling 51:

$$\begin{aligned}
x \Rightarrow 0 &\equiv \langle \text{st36} \rangle \neg x \vee 0 \\
&\equiv \langle \text{st15} \rangle \neg x
\end{aligned}$$

Stelling 52:

$$\begin{aligned}
0 \Rightarrow x &\equiv \langle \text{st36} \rangle \neg 0 \vee x \\
&\equiv \langle \text{st7} \rangle 1 \vee x
\end{aligned}$$

$$\begin{aligned} &\equiv \langle \text{ax7} \rangle x \vee 1 \\ &\equiv \langle \text{st14} \rangle 1 \end{aligned}$$

Stelling 53: a:

$$\begin{aligned} x \Rightarrow x \vee y &\equiv \langle \text{st36} \rangle \neg x \vee (x \vee y) \\ &\equiv \langle \text{ax8} \rangle (\neg x \vee x) \vee y \\ &\equiv \langle \text{ax7} \rangle (x \vee \neg x) \vee y \\ &\equiv \langle \text{ax11} \rangle 1 \vee y \\ &\equiv \langle \text{ax7} \rangle y \vee 1 \\ &\equiv \langle \text{st14} \rangle 1 \quad (\text{st2}) \end{aligned}$$

Stelling 53: b:

$$\begin{aligned} x \wedge y \Rightarrow x &\equiv \langle \text{st36} \rangle \neg (x \wedge y) \vee x \\ &\equiv \langle \text{st29a} \rangle (\neg x \vee \neg y) \vee x \\ &\equiv \langle \text{ax7} \rangle (\neg y \vee \neg x) \vee x \\ &\equiv \langle \text{ax8} \rangle \neg y \vee (\neg x \vee x) \\ &\equiv \langle \text{ax7} \rangle \neg y \vee (x \vee \neg x) \\ &\equiv \langle \text{ax11} \rangle \neg y \vee 1 \\ &\equiv \langle \text{st14} \rangle 1 \quad (\text{st2}) \end{aligned}$$

Stelling 53: c:

$$\begin{aligned} x \wedge y \Rightarrow x \vee y &\equiv \langle \text{st36} \rangle \neg (x \wedge y) \vee (x \vee y) \\ &\equiv \langle \text{st29a} \rangle (\neg x \vee \neg y) \vee (x \vee y) \\ &\equiv \langle \text{st16} \rangle ((\neg x \vee \neg y) \vee x) \vee ((\neg x \vee \neg y) \vee y) \\ &\equiv \langle \text{ax7, st16} \rangle (x \vee \neg x) \vee (x \vee \neg y) \vee (y \vee \neg x) \vee (y \vee \neg y) \\ &\equiv \langle \text{ax7, ax11} \rangle 1 \vee (x \vee \neg y) \vee (y \vee \neg x) \vee 1 \\ &\equiv \langle \text{st14} \rangle 1 \vee 1 \\ &\equiv \langle \text{st14} \rangle 1 \quad (\text{st2}) \end{aligned}$$

Stelling 53: d:

$$\begin{aligned} x \vee (y \wedge z) \Rightarrow x \vee y &\equiv \langle \text{st27} \rangle (x \vee y) \wedge (x \vee z) \Rightarrow x \vee y \\ &\equiv \langle \text{st36} \rangle \neg ((x \vee y) \wedge (x \vee z)) \vee (x \vee y) \\ &\equiv \langle \text{st29a} \rangle \neg (x \vee y) \vee \neg (x \vee z) \vee (x \vee y) \\ &\equiv \langle \text{ax7} \rangle \neg (x \vee y) \vee (x \vee y) \vee \neg (x \vee z) \\ &\equiv \langle \text{ax7} \rangle (x \vee y) \vee \neg (x \vee y) \vee \neg (x \vee z) \\ &\equiv \langle \text{ax11} \rangle 1 \vee \neg (x \vee z) \\ &\equiv \langle \text{ax7} \rangle \neg (x \vee z) \vee 1 \\ &\equiv \langle \text{st14} \rangle 1 \quad (\text{st2}) \end{aligned}$$

Stelling 53: e:

$$\begin{aligned} x \wedge y \Rightarrow x \wedge (y \vee z) &\equiv \langle \text{st28} \rangle (x \wedge y) \Rightarrow ((x \wedge y) \vee (x \wedge z)) \\ &\equiv \langle \text{st36} \rangle \neg (x \wedge y) \vee ((x \wedge y) \vee (x \wedge z)) \\ &\equiv \langle \text{ax8} \rangle (\neg (x \wedge y) \vee (x \wedge y)) \vee (x \wedge z) \\ &\equiv \langle \text{ax7} \rangle ((x \wedge y) \vee \neg (x \wedge y)) \vee (x \wedge z) \\ &\equiv \langle \text{ax11} \rangle 1 \vee (x \wedge z) \end{aligned}$$

$$\begin{aligned} &\equiv \langle \text{ax7} \rangle (x \wedge z) \vee 1 \\ &\equiv \langle \text{st14} \rangle 1 \quad (\text{st2}) \end{aligned}$$

Stelling 54:

$$\begin{aligned} x \wedge (x \Rightarrow y) \Rightarrow y &\equiv \langle \text{st43} \rangle (x \wedge y) \Rightarrow y \\ &\equiv \langle \text{st18} \rangle (y \wedge x) \Rightarrow y \\ &\equiv \langle \text{instantiatie, [y, x := x, y]} \rangle x \wedge y \Rightarrow x \end{aligned}$$

Stelling 55:

$$\begin{aligned} (x \Rightarrow z) \wedge (y \Rightarrow z) &\equiv \langle \text{st36} \rangle (\neg x \vee z) \wedge (\neg y \vee z) \\ &\equiv \langle \text{ax7} \rangle (z \vee \neg x) \wedge (z \vee \neg y) \\ &\equiv \langle \text{st27} \rangle z \vee (\neg x \wedge \neg y) \\ &\equiv \langle \text{st29b} \rangle z \vee (\neg (x \vee y)) \\ &\equiv \langle \text{ax7} \rangle \neg (x \vee y) \vee z \\ &\equiv \langle \text{st36} \rangle (x \vee y) \Rightarrow z \end{aligned}$$

Stelling 56:

$$\begin{aligned} (x \Rightarrow z) \wedge (\neg x \Rightarrow z) &\equiv \langle \text{st55} \rangle (x \vee \neg x) \Rightarrow z \\ &\equiv \langle \text{ax11} \rangle 1 \Rightarrow z \\ &\equiv \langle \text{st50} \rangle z \end{aligned}$$

Stelling 57:

$$\begin{aligned} (x \Rightarrow y) \wedge (y \Rightarrow x) &\equiv \langle \text{st36} \rangle (\neg x \vee y) \wedge (\neg y \vee x) \\ &\equiv \langle \text{st28} \rangle ((\neg x \vee y) \wedge \neg y) \vee ((\neg x \vee y) \wedge x) \\ &\equiv \langle \text{st18} \rangle ((\neg x \vee y) \wedge \neg y) \vee (x \wedge (\neg x \vee y)) \\ &\equiv \langle \text{st26a} \rangle ((\neg x \vee y) \wedge \neg y) \vee (x \wedge y) \\ &\equiv \langle \text{st18, ax7, st6} \rangle (\neg y \wedge (\neg \neg y \vee \neg x)) \vee (x \wedge y) \\ &\equiv \langle \text{st26a} \rangle (\neg y \wedge \neg x) \vee (x \wedge y) \\ &\equiv \langle \text{ax7, st18} \rangle (x \wedge y) \vee (\neg x \wedge \neg y) \\ &\equiv \langle \text{st34} \rangle x \equiv y \end{aligned}$$

Stelling 58:

$$\begin{aligned} ((x \Rightarrow y) \wedge (y \Rightarrow x)) \Rightarrow (x \equiv y) &\equiv \langle \text{st57} \rangle (x \equiv y) \Rightarrow (x \equiv y) \\ &\equiv \langle \text{st48} \rangle 1 \quad (\text{st2}) \end{aligned}$$

Stelling 59: (a):

Stelling 60:

Stelling 61:

Stelling 62:

Stelling 63:

Stelling 64: