

Hoofdstuk 5: Predikatenrekenen:

Oef 1: (a):

$l(f(u),w)$

$l(w,w)$

z

Oef 1: (b):

$g(x) \rightarrow$ kan niet, zou $g(w)$ moeten zijn

Oef 1: (c):

$l(f(h(x)),f(u))$

$l(f(u),f(u))$

$l(w,w)$

z

Oef 1: (d):

$f(h(g(f(y)))) \rightarrow$ kan niet, zou $f(u)$ moeten zijn

Oef 1: (e):

$k(h(x),h(x))$

$k(u,u) \rightarrow$ kan niet, zou $k(x,u)$ moeten zijn

Oef 2: (a):

$*(x \mid 0 \leq x + r < n \cdot x + v) [v := 3]$

$*(x \mid 0 \leq x + r < n \cdot x + 3)$

Oef 2: (b):

$*(x \mid 0 \leq x + r < n \cdot x + v) [x := 3]$

$*(x \mid 0 \leq x + r < n \cdot x + v)$

Oef 2: (c):

$*(x \mid 0 \leq x + r < n \cdot x + v) [n := n + x]$

$*(y \mid (0 \leq x + r < n) [x := y][n := n + x] \cdot (x + v) [x := y][n := n + x])$

$*(y \mid 0 \leq y + r < (n + x) \cdot y + v)$

Oef 2: (d):

$*(x \mid 0 \leq x + r < n \cdot *(y \mid 0 \leq y \cdot x + y + n)) [n := x + y]$

$*(z \mid (0 \leq x + r < n) [x := z][n := x + y] \cdot (*(y \mid 0 \leq y \cdot x + y + n)) [x := z][n := x + y])$

$*(z \mid (0 \leq z + r < (x + y)) \cdot *(u \mid (0 \leq y) [y := u][n := x + y] \cdot (z + y + n) [y := u][n := x + y]))$

$*(z \mid (0 \leq z + r < (x + y)) \cdot *(u \mid (0 \leq u) \cdot (z + u + x + y)))$

Oef 2: (e):

$$\begin{aligned}
&*(x \mid 0 \leq x + r < n . *(y \mid 0 \leq y . x + y + n)) [r := y] \\
&*(x \mid 0 \leq x + y < n . (*(y \mid 0 \leq y . x + y + n)) [r := y]) \\
&*(x \mid 0 \leq x + y < n . *(y \mid 0 \leq y . x + y + n))
\end{aligned}$$

Oef 3:

Lemma:

$$\begin{aligned}
&(e = y) \wedge d[v := e] \equiv (e = y) \wedge d[v := y] \\
&\equiv \langle \text{st31} \rangle (e = y) \wedge (d[v := e] \equiv d[v := y]) \equiv (e = y) \\
&\equiv \langle \text{st37} \rangle (e = y) \Rightarrow (d[v := e] \equiv d[v := y]) \\
&\rightarrow \text{aanname van het antecedent } (e = y) + \text{Leibniz}
\end{aligned}$$

$ \begin{aligned} *(y \mid r[x := f(y)] . e[x := f(y)]) &\equiv \langle \text{eenpuntsregel} \rangle \\ &\equiv \langle \text{vernesteling} \rangle \\ &\equiv \langle \text{lemma} \rangle \\ &\equiv \langle \text{subst.} \rangle \\ &\equiv \langle \text{vernesteling} \rangle \\ &\equiv \langle \text{aanname} \rangle \\ &\equiv \langle \text{eenpuntsregel} \rangle \\ &\equiv \langle \text{subst. } y \text{ niet vrij in } e \rangle \end{aligned} $	$ \begin{aligned} &*(y \mid r[x := f(y)] . *(x \mid x := f(y) . e)) \\ &*(x, y \mid r[x := f(y)] \wedge (x := f(y)) . e) \\ &*(x, y \mid r[x := x] \wedge (x := f(y)) . e) \\ &*(x, y \mid r \wedge (x = f(y)) . e) \\ &*(x \mid r . *(y \mid x = f(y) . e)) \\ &*(x \mid r . *(y \mid f^1(x) = y . e)) \\ &*(x \mid r . e[y := f^1(x)]) \\ &*(x \mid r . e) \end{aligned} $
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Stelling 65: (a):

$$\begin{aligned}
\forall (x \mid r . p) &\equiv \langle \text{ax23} \rangle \forall (x . r \Rightarrow p) \\
&\equiv \langle \text{st36} \rangle \forall (x . \neg r \vee p)
\end{aligned}$$

Stelling 65: (b):

$$\begin{aligned}
\forall (x \mid r . p) &\equiv \langle \text{ax23} \rangle \forall (x . r \Rightarrow p) \\
&\equiv \langle \text{st37} \rangle \forall (x . r \wedge p \equiv r)
\end{aligned}$$

Stelling 65: (c):

$$\begin{aligned}
\forall (x \mid r . p) &\equiv \langle \text{ax23} \rangle \forall (x . r \Rightarrow p) \\
&\equiv \langle \text{ax13} \rangle \forall (x . r \vee p \equiv p)
\end{aligned}$$

Stelling 66: (a):

$$\begin{aligned}
\forall (x \mid q \wedge r . p) &\equiv \langle \text{ax23} \rangle \forall (x . q \wedge r \Rightarrow p) \\
&\equiv \langle \text{st42} \rangle \forall (x . q \Rightarrow (r \Rightarrow p)) \\
&\equiv \langle \text{ax13} \rangle \forall (x \mid q . r \Rightarrow p)
\end{aligned}$$

Stelling 66: (b):

$$\begin{aligned}
\forall (x \mid q \wedge r . p) &\equiv \langle \text{ax23} \rangle \forall (x . q \wedge r \Rightarrow p) \\
&\equiv \langle \text{st42} \rangle \forall (x . q \Rightarrow (r \Rightarrow p)) \\
&\equiv \langle \text{st36} \rangle \forall (x . q \Rightarrow (\neg r \vee p)) \\
&\equiv \langle \text{ax23} \rangle \forall (x \mid q . \neg r \vee p)
\end{aligned}$$

Stelling 66: (c):

$$\begin{aligned}\forall (x \mid q \wedge r . p) &\equiv \langle \text{ax23} \rangle \forall (x . q \wedge r \Rightarrow p) \\ &\equiv \langle \text{st42} \rangle \forall (x . q \Rightarrow (r \Rightarrow p)) \\ &\equiv \langle \text{st37} \rangle \forall (x . q \Rightarrow (r \wedge p \equiv r)) \\ &\equiv \langle \text{ax23} \rangle \forall (x \mid q . r \wedge p \equiv r)\end{aligned}$$

Stelling 66: (d):

$$\begin{aligned}\forall (x \mid q \wedge r . p) &\equiv \langle \text{ax23} \rangle \forall (x . q \wedge r \Rightarrow p) \\ &\equiv \langle \text{st42} \rangle \forall (x . q \Rightarrow (r \Rightarrow p)) \\ &\equiv \langle \text{ax13} \rangle \forall (x . q \Rightarrow (r \vee p \equiv p)) \\ &\equiv \langle \text{ax23} \rangle \forall (x \mid q . r \vee p \equiv p)\end{aligned}$$

Stelling 67:

$$\begin{aligned}\forall (x \mid r . p) &\equiv \langle \text{st15} \rangle \forall (x \mid r . p \vee 0) \\ &\equiv \langle \text{ax24} \rangle p \vee \forall (x \mid r . 0) \\ &\equiv \langle \text{ax23} \rangle p \vee \forall (x . r \Rightarrow 0) \\ &\equiv \langle \text{st51} \rangle p \vee \forall (x . \neg r)\end{aligned}$$

Stelling 68:

$$\neg \forall (x . \neg r) \Rightarrow (\forall (x \mid r . p \wedge q) \equiv p \wedge \forall (x \mid r . q))$$

$$\begin{aligned}\forall (x \mid r . p \wedge q) &\equiv \langle \text{ax18} \rangle \forall (x \mid r . p) \wedge \forall (x \mid r . q) \\ &\equiv \langle \text{st67} \rangle (p \vee \forall (x . \neg r)) \wedge \forall (x \mid r . q) \\ &\equiv \langle \text{st6} \rangle (p \vee \neg \forall (x . \neg r)) \wedge \forall (x \mid r . q) \\ &\equiv \langle \neg \forall (x . \neg r) \equiv 1 \rangle (p \vee \neg 1) \wedge \forall (x \mid r . q) \\ &\equiv \langle \text{ax4, st15} \rangle p \wedge \forall (x \mid r . q)\end{aligned}$$

Stelling 69:

$$\begin{aligned}\forall (x \mid r . 1) &\equiv \langle \text{ax23} \rangle \forall (x . r \Rightarrow 1) \\ &\equiv \langle \text{st49} \rangle \forall (x . 1) \\ &\equiv \langle \text{st52} \rangle \forall (x . 0 \Rightarrow x) \\ &\equiv \langle \text{ax23} \rangle \forall (x \mid 0 . x) \\ &\equiv \langle \text{ax16a} \rangle 1\end{aligned}$$

Stelling 70:

$$\begin{aligned}\forall (x . p \equiv q) \Rightarrow (\forall (x \mid r . p) \equiv \forall (x \mid r . q)) \\ &\equiv \forall (x \mid r . p \equiv q) \Rightarrow (\forall (x \mid r . p) \equiv \forall (x \mid r . q)) \\ &\equiv \langle \text{st39} \rangle \forall (x \mid r . p \equiv q) \wedge \forall (x \mid r . p) \equiv \forall (x \mid r . p \equiv q) \wedge \forall (x \mid r . q) \\ &\equiv \langle \text{ax18a} \rangle \forall (x \mid r . (p \equiv q) \wedge p) \equiv \forall (x \mid r . (p \equiv q) \wedge q) \\ &\equiv \langle \text{st60a} \rangle \forall (x \mid r . (p \equiv q) \wedge q) \equiv \forall (x \mid r . (p \equiv q) \wedge q) \\ &\rightarrow \text{inst. van st3}\end{aligned}$$

Stelling 71:

$$\begin{aligned}\forall (x \mid q \vee r . p) &\equiv \langle \text{ax19a} \rangle \forall (x \mid q . p) \wedge \forall (x \mid r . p) \\ &\Rightarrow \langle \text{st53b} \rangle \forall (x \mid q . p)\end{aligned}$$

Stelling 72:

$$\begin{aligned}\forall (x \mid r . p \wedge q) &\equiv \langle \text{ax18a} \rangle \forall (x \mid r . p) \wedge \forall (x \mid r . q) \\ &\Rightarrow \langle \text{st53b} \rangle \forall (x \mid r . p)\end{aligned}$$

Stelling 73:

$$\begin{aligned}\forall (x \mid r . q \Rightarrow p) &\Rightarrow (\forall (x \mid r . q) \Rightarrow \forall (x \mid r . p)) \\ &\equiv \langle \text{st42} \rangle \forall (x \mid r . q \Rightarrow p) \Rightarrow (\forall (x \mid r . q) \Rightarrow \forall (x \mid r . p)) \\ &\equiv \langle \text{ax18a} \rangle \forall (x \mid r . (q \Rightarrow p) \wedge q) \Rightarrow \forall (x \mid r . p) \\ &\equiv \langle \text{st37} \rangle \forall (x \mid r . (q \Rightarrow p) \wedge q) \wedge \forall (x \mid r . p) \equiv \forall (x \mid r . (q \Rightarrow p) \wedge q) \\ &\equiv \langle \text{ax18a} \rangle \forall (x \mid r . (q \Rightarrow p) \wedge q \wedge p) \equiv \forall (x \mid r . (q \Rightarrow p) \wedge q) \\ &\equiv \langle \text{st43} \rangle \forall (x \mid r . q \wedge p \wedge p) \equiv \forall (x \mid r . q \wedge p) \\ &\equiv \langle \text{st20} \rangle \forall (x \mid r . q \wedge p) \equiv \forall (x \mid r . q \wedge p) \\ &\rightarrow \text{inst. van st3}\end{aligned}$$

Stelling 74:

$$\begin{aligned}\forall (x . p) &\equiv \langle \text{ax22}, y \notin \varphi p, y \notin \varphi e \rangle \forall (y . p[x := y]) \\ &\equiv \langle \text{ax11} \rangle \forall (y \mid y = e \vee \neg(y = e) . p[x := y]) \\ &\Rightarrow \langle \text{st71} \rangle \forall (y \mid y = e . p[x := y]) \\ &\equiv \langle \text{ax17} \rangle p[x := y] [y := e] \\ &\equiv \langle \text{subst} \rangle p[x := e]\end{aligned}$$

Stelling 75: (a):

$$\begin{aligned}\forall (x \mid r . p) &\equiv \langle \text{st6} \rangle \neg \neg \forall (x \mid r . \neg \neg p) \\ &\equiv \langle \text{ax25} \rangle \neg \exists (x \mid r . \neg p)\end{aligned}$$

Stelling 75: (b):

$$\begin{aligned}\exists (x \mid r . \neg p) &\equiv \langle \text{ax25} \rangle \neg \forall (x \mid r . \neg \neg p) \\ &\equiv \langle \text{ax6} \rangle \neg \forall (x \mid r . p)\end{aligned}$$

Stelling 75: (c):

$$\begin{aligned}\forall (x \mid r . \neg p) &\equiv \langle \text{st6} \rangle \neg \neg \forall (x \mid r . \neg p) \\ &\equiv \langle \text{ax25} \rangle \neg \exists (x \mid r . p)\end{aligned}$$

Stelling 76: (a):

$$\begin{aligned}\exists (x \mid r . p) &\equiv \langle \text{ax25} \rangle \neg \forall (x \mid r . \neg p) \\ &\equiv \langle \text{st65a} \rangle \neg \forall (x . \neg r \vee \neg p) \\ &\equiv \langle \text{st29a} \rangle \neg \forall (x . \neg(r \wedge p)) \\ &\equiv \langle \text{ax25} \rangle \exists (x . r \wedge p)\end{aligned}$$

Stelling 76: (b):

$$\begin{aligned}\exists (x | q \wedge r . p) &\equiv \langle \text{ax25} \rangle \neg \forall (x | q \wedge r . \neg p) \\ &\equiv \langle \text{ax23} \rangle \neg \forall (x . (q \wedge r) \Rightarrow \neg p) \\ &\equiv \langle \text{st42} \rangle \neg \forall (x . (q \Rightarrow (r \Rightarrow \neg p))) \\ &\equiv \langle \text{st36} \rangle \neg \forall (x . (q \Rightarrow (\neg r \vee \neg p))) \\ &\equiv \langle \text{st29a} \rangle \neg \forall (x . (q \Rightarrow (\neg(r \wedge p)))) \\ &\equiv \langle \text{ax23} \rangle \neg \forall (x | q . \neg(r \wedge p)) \\ &\equiv \langle \text{ax25} \rangle \exists (x | q . r \wedge p)\end{aligned}$$

Stelling 77:

$$\begin{aligned}q \wedge \exists (x | r . p) &\equiv \langle \text{st6} \rangle \neg \neg (q \wedge \exists (x | r . p)) \\ &\equiv \langle \text{st29a} \rangle \neg (\neg q \vee \neg \exists (x | r . p)) \\ &\equiv \langle \text{st75c} \rangle \neg (\neg q \vee \forall (x | r . \neg p)) \\ &\equiv \langle \text{ax24} \rangle \neg \forall (x | r . \neg q \vee \neg p) \\ &\equiv \langle \text{st29a} \rangle \neg \forall (x | r . \neg (q \wedge p)) \\ &\equiv \langle \text{ax25} \rangle \exists (x | r . q \wedge p)\end{aligned}$$

Stelling 78:

$$\begin{aligned}p \wedge \exists (x . r) &\equiv \langle \text{st20} \rangle p \wedge \exists (x | r \wedge r) \\ &\equiv \langle \text{st76a} \rangle p \wedge \exists (x | r . r) \\ &\equiv \langle \text{st77} \rangle \exists (x | r . p \wedge r) \\ &\equiv \langle \text{st76a} \rangle \exists (x . r \wedge p \wedge r) \\ &\equiv \langle \text{st20} \rangle \exists (x . r \wedge p) \\ &\equiv \langle \text{st76a} \rangle \exists (x | r . p)\end{aligned}$$

Stelling 79:

$$\begin{aligned}\exists (x . r) &\equiv \langle \text{ax25} \rangle \neg \forall (x . \neg r) \\ &\Rightarrow \langle \text{st68} \rangle \forall (x | r . \neg p \wedge \neg q) \equiv \neg p \wedge \forall (x | r . \neg q) \\ &\equiv \langle \text{st29b} \rangle \forall (x | r . \neg (p \vee q)) \equiv \neg p \wedge \forall (x | r . \neg q) \\ &\equiv \langle \text{st75c} \rangle \neg \exists (x | r . p \vee q) \equiv \neg p \wedge \neg \exists (x | r . q) \\ &\equiv \langle \text{st29b} \rangle \neg \exists (x | r . p \vee q) \equiv \neg (p \vee \exists (x | r . q)) \\ &\equiv \langle \text{st5} \rangle \exists (x | r . p \vee q) \equiv p \vee \exists (x | r . q)\end{aligned}$$

Stelling 80:

$$\begin{aligned}\exists (x | r . 0) &\equiv \langle \text{st76a} \rangle \exists (x . r \wedge 0) \\ &\equiv \langle \text{st18} \rangle \exists (x . 0 \wedge r) \\ &\equiv \langle \text{st76a} \rangle \exists (x | 0 . r) \\ &\equiv \langle \text{ax16b} \rangle 0\end{aligned}$$

Stelling 81:

$$\begin{aligned}\exists (x | r . p) \Rightarrow \exists (x | q \vee r . p) & \\ \equiv \langle \text{ax19b} \rangle \exists (x | r . p) \Rightarrow \exists (x | q . p) \vee \exists (x | r . p) & \quad (\text{st53a})\end{aligned}$$

Stelling 82:

$$\begin{aligned}\exists (x | r . p) \Rightarrow \exists (x | r . p \vee q) \\ \equiv \langle \text{ax18b} \rangle \exists (x | r . p) \Rightarrow \exists (x | r . p) \vee \exists (x | r . q) \quad (\text{st53a})\end{aligned}$$

Stelling 83:

$$\begin{aligned}\forall (x | r . q \Rightarrow p) &\equiv \langle \text{st38} \rangle \forall (x | r . \neg p \Rightarrow \neg q) \\ &\Rightarrow \langle \text{st73} \rangle \forall (x | r . \neg p) \Rightarrow \forall (x | r . \neg q) \\ &\equiv \langle \text{st75c} \rangle \neg \exists (x | r . p) \Rightarrow \neg \exists (x | r . q) \\ &\equiv \langle \text{st38} \rangle \exists (x | r . q) \Rightarrow \exists (x | r . p)\end{aligned}$$

Stelling 84:

$$\begin{aligned}p[x := e] \Rightarrow \exists (x . p) \\ \equiv \langle \text{st38} \rangle \neg \exists (x . p) \Rightarrow \neg(p[x := e]) \\ \equiv \langle \text{st75c} \rangle \forall (x . \neg p) \Rightarrow \neg(p[x := e]) \\ \equiv \langle \text{subst} \rangle \forall (x . \neg p) \Rightarrow \neg(p)[x := e] \\ \rightarrow \text{inst v st74}\end{aligned}$$

Stelling 85:

$$\begin{aligned}\exists (x | r . \forall (y | q . p)) &\equiv \langle \text{st75c} \rangle \neg \forall (x | r . \neg \forall (y | q . p)) \\ &\equiv \langle \text{st74, st84} \rangle \neg \exists (x | r . \neg \forall (y | q . p)) \\ &\equiv \langle \text{st74, st84} \rangle \neg \exists (x | r . \neg \exists (y | q . p)) \\ &\equiv \langle \text{st75a} \rangle \forall (y | q . \exists (x | r . p))\end{aligned}$$

Stelling 86:

$$\begin{aligned}\forall (x | r . p) \vee \forall (x | r . q) \Rightarrow \forall (x | r . p \vee q) \\ \equiv \langle \text{st25a} \rangle (\forall (x | r . p \wedge (p \vee q)) \vee \forall (x | r . q \wedge (p \vee q)) \Rightarrow \forall (x | r . p \vee q) \\ \equiv \langle \text{st55} \rangle (\forall (x | r . p \wedge (p \vee q)) \Rightarrow \forall (x | r . p \vee q)) \\ \quad \wedge (\forall (x | r . q \wedge (p \vee q)) \Rightarrow \forall (x | r . p \vee q)) \\ \equiv \langle \text{st72} \rangle 1 \wedge 1 \\ \equiv \langle \text{st20} \rangle 1\end{aligned}$$

Stelling 87:

$$\begin{aligned}\exists (x | r . p \wedge q) &\equiv \langle \text{st75b} \rangle \neg \forall (x | r . \neg(p \wedge q)) \\ &\equiv \langle \text{st29a} \rangle \neg \forall (x | r . \neg p \vee \neg q) \\ &\Rightarrow \langle \text{st86} \rangle \neg(\forall (x | r . \neg p) \vee \forall (x | r . \neg q)) \\ &\equiv \langle \text{st29b} \rangle \neg \forall (x | r . \neg p) \wedge \neg \forall (x | r . \neg q) \\ &\equiv \langle \text{st75c} \rangle \exists (x | r . p) \wedge \exists (x | r . q)\end{aligned}$$