Hoofdstuk 5: Predikatenrekenen:

Oef 1: (a):

l(f(u),w)
l(w,w)
z

Oef 1: (b):

g(x) -> kan niet, zou g(w) moeten zijn

Oef 1: (c):

l(f(h(x)),f(u))
l(f(u),f(u))
l(w,w)
z

Oef 1: (d):

f(h(g(f(y)))) -> kan niet, zou f(u) moeten zijn

Oef 1: (e):

k(h(x),h(x))
k(u,u) -> kan niet, zou k(x,u) moeten zijn

Oef 2: (a):

$*(x \mid 0 \leq x + r < n \,.\, x + v)\ [v := 3]$
$*(x \mid 0 \leq x + r < n \,.\, x + 3)$

Oef 2: (b):

$*(x \mid 0 \leq x + r < n \,.\, x + v)\ [x := 3]$
$*(x \mid 0 \leq x + r < n \,.\, x + v)$

Oef 2: (c):

$*(x \mid 0 \leq x + r < n \,.\, x + v)\ [n := n + x]$
$*(y \mid (0 \leq x + r < n)\ [x := y][n := n + x] \,.\, (x + v)\ [x := y][n := n + x])$
$*(y \mid 0 \leq y + r < (n + x) \,.\, y + v)$

Oef 2: (d):

$*(x \mid 0 \leq x + r < n \,.\, *(y \mid 0 \leq y \,.\, x + y + n))\ [n := x + y]$
$*(z \mid (0 \leq x + r < n)\ [x := z][n := x + y] \,.\, (*(y \mid 0 \leq y \,.\, x + y + n))\ [x := z][n := x + y])$
$*(z \mid (0 \leq z + r < (x + y)) \,.\, *(u \mid (0 \leq y)\ [y := u][n := x + y] \,.\, (z + y + n)\ [y := u][n := x + y]))$
$*(z \mid (0 \leq z + r < (x + y)) \,.\, *(u \mid (0 \leq u) \,.\, (z + u + x + y)))$

<u>Oef 2: (e):</u>

*(x | 0 ≤ x + r < n . *(y | 0 ≤ y . x + y + n)) [r := y]
(x | 0 ≤ x + y < n . ((y | 0 ≤ y . x + y + n)) [r := y])
*(x | 0 ≤ x + y < n . *(y | 0 ≤ y . x + y + n))

<u>Oef 3:</u>

Lemma:
(e = y) ˄ d[v := e] ≡ (e = y) ˄ d[v := y]
≡ <st31> (e = y) ˄ (d[v := e] ≡ d[v := y]) ≡ (e = y)
≡ <st37> (e = y) => (d[v := e] ≡ d[v := y])
→ aanname van het antecedent (e = y) + Leibniz

*(y | r[x := f(y)] . e[x := f(y)] ≡ <eenpuntsregel> *(y | r[x := f(y)] . *(x | x := f(y) . e))
≡ <vernesteling> *(x,y | r[x := f(y)] ˄ (x := f(y)) . e)
≡ <lemma> *(x,y | r[x := x] ˄ (x := f(y)) . e)
≡ <subst.> *(x,y | r ˄ (x = f(y)) . e)
≡ <vernesteling> *(x | r . *(y | x = f(y) . e))
≡ <aanname> *(x | r . *(y | $f^{-1}(x)$ = y . e))
≡ <eenpuntsregel> *(x | r . e[y := $f^{-1}(x)$)
≡ <subst. y niet vrij in e> *(x | r . e)

<u>Stelling 65: (a):</u>

∀ (x | r . p) ≡ <ax23> ∀ (x . r => p)
≡ <st36> ∀ (x . ¬r ˅ p)

<u>Stelling 65: (b):</u>

∀ (x | r . p) ≡ <ax23> ∀ (x . r => p)
≡ <st37> ∀ (x . r ˄ p ≡ r)

<u>Stelling 65: (c):</u>

∀ (x | r . p) ≡ <ax23> ∀ (x . r => p)
≡ <ax13> ∀ (x . r ˅ p ≡ p)

<u>Stelling 66: (a):</u>

∀ (x | q ˄ r . p)≡ <ax23> ∀ (x . q ˄ r => p)
≡ <st42> ∀ (x . q => (r => p))
≡ <ax13> ∀ (x | q . r => p)

<u>Stelling 66: (b):</u>

∀ (x | q ˄ r . p)≡ <ax23> ∀ (x . q ˄ r => p)
≡ <st42> ∀ (x . q => (r => p))
≡ <st36> ∀ (x . q => (¬r ˅ p))
≡ <ax23> ∀ (x | q . ¬r ˅ p)

Stelling 66: (c):

$\forall (x \mid q \wedge r \,.\, p) \equiv \langle ax23 \rangle\ \forall (x \,.\, q \wedge r \Rightarrow p)$
$\equiv \langle st42 \rangle\ \forall (x \,.\, q \Rightarrow (r \Rightarrow p))$
$\equiv \langle st37 \rangle\ \forall (x \,.\, q \Rightarrow (r \wedge p \equiv r))$
$\equiv \langle ax23 \rangle\ \forall (x \mid q \,.\, r \wedge p \equiv r)$

Stelling 66: (d):

$\forall (x \mid q \wedge r \,.\, p) \equiv \langle ax23 \rangle\ \forall (x \,.\, q \wedge r \Rightarrow p)$
$\equiv \langle st42 \rangle\ \forall (x \,.\, q \Rightarrow (r \Rightarrow p))$
$\equiv \langle ax13 \rangle\ \forall (x \,.\, q \Rightarrow (r \vee p \equiv p))$
$\equiv \langle ax23 \rangle\ \forall (x \mid q \,.\, r \vee p \equiv p)$

Stelling 67:

$\forall (x \mid r \,.\, p) \equiv \langle st15 \rangle\ \forall (x \mid r \,.\, p \vee 0)$
$\equiv \langle ax24 \rangle\ p \vee \forall (x \mid r \,.\, 0)$
$\equiv \langle ax23 \rangle\ p \vee \forall (x \,.\, r \Rightarrow 0)$
$\equiv \langle st51 \rangle\ p \vee \forall (x \,.\, \neg r)$

Stelling 68:

$\neg \forall (x \,.\, \neg r) \Rightarrow (\forall (x \mid r \,.\, p \wedge q) \equiv p \wedge \forall (x \mid r \,.\, q))$

$\forall (x \mid r \,.\, p \wedge q) \equiv \langle ax18 \rangle\ \forall (x \mid r \,.\, p) \wedge \forall (x \mid r \,.\, q)$
$\equiv \langle st67 \rangle\ (p \vee \forall (x \,.\, \neg r)) \wedge \forall (x \mid r \,.\, q)$
$\equiv \langle st6 \rangle\ (p \vee \neg\neg \forall (x \,.\, \neg r)) \wedge \forall (x \mid r \,.\, q)$
$\equiv \langle \neg \forall (x \,.\, \neg r) \equiv 1 \rangle\ (p \vee \neg 1) \wedge \forall (x \mid r \,.\, q)$
$\equiv \langle ax4, st15 \rangle\ p \wedge \forall (x \mid r \,.\, q))$

Stelling 69:

$\forall (x \mid r \,.\, 1) \equiv \langle ax23 \rangle\ \forall (x \,.\, r \Rightarrow 1)$
$\equiv \langle st49 \rangle\ \forall (x \,.\, 1)$
$\equiv \langle st52 \rangle\ \forall (x \,.\, 0 \Rightarrow x)$
$\equiv \langle ax23 \rangle\ \forall (x \mid 0 \,.\, x)$
$\equiv \langle ax16a \rangle\ 1$

Stelling 70:

$\forall (x \,.\, p \equiv q) \Rightarrow (\forall (x \mid r \,.\, p) \equiv \forall (x \mid r \,.\, q))$
$\equiv \forall (x \mid r \,.\, p \equiv q) \Rightarrow (\forall (x \mid r \,.\, p) \equiv \forall (x \mid r \,.\, q))$
$\equiv \langle st39 \rangle \forall (x \mid r \,.\, p \equiv q) \wedge \forall (x \mid r \,.\, p) \equiv \forall (x \mid r \,.\, p \equiv q) \wedge \forall (x \mid r \,.\, q)$
$\equiv \langle ax18a \rangle \forall (x \mid r \,.\, (p \equiv q) \wedge p) \equiv \forall (x \mid r \,.\, (p \equiv q) \wedge q)$
$\equiv \langle st60a \rangle \forall (x \mid r \,.\, (p \equiv q) \wedge q) \equiv \forall (x \mid r \,.\, (p \equiv q) \wedge q)$
→ inst. van st3

Stelling 71:

∀ (x | q ∨ r . p) ≡ <ax19a> ∀ (x | q . p) ∧ ∀ (x | r . p)
=> <st53b> ∀ (x | q . p)

Stelling 72:

∀ (x | r . p ∧ q) ≡ <ax18a> ∀ (x | r . p) ∧ ∀ (x | r . q)
=> <st53b> ∀ (x | r . p)

Stelling 73:

∀ (x | r . q => p) => (∀ (x | r . q) => ∀ (x | r . p))
≡ <st42> ∀ (x | r . q => p) => (∀ (x | r . q) => ∀ (x | r . p))
≡ <ax18a> ∀ (x | r . (q => p) ∧ q) => ∀ (x | r . p)
≡ <st37> ∀ (x | r . (q => p) ∧ q) ∧ ∀ (x | r . p) ≡ ∀ (x | r . (q => p) ∧ q)
≡ <ax18a> ∀ (x | r . (q => p) ∧ q ∧ p) ≡ ∀ (x | r . (q => p) ∧ q)
≡ <st43> ∀ (x | r . q ∧ p ∧ p) ≡ ∀ (x | r . q ∧ p)
≡ <st20> ∀ (x | r . q ∧ p) ≡ ∀ (x | r . q ∧ p)
→ inst. van st3

Stelling 74:

∀ (x . p) ≡ <ax22, y ∉ φp, y ∉ φe> ∀ (y . p[x := y])
≡ <ax11> ∀ (y | y = e ∨ ¬(y = e) . p[x := y])
=> <st71> ∀ (y | y = e . p[x := y])
≡ <ax17> p[x := y] [y := e]
≡ <subst> p[x := e]

Stelling 75: (a):

∀ (x | r . p) ≡ <st6> ¬¬∀ (x | r . ¬¬p)
≡ <ax25> ¬∃ (x | r . ¬p)

Stelling 75: (b):

∃ (x | r . ¬p) ≡ <ax25> ¬∀ (x | r . ¬¬p)
≡ <ax6> ¬∀ (x | r . p)

Stelling 75: (c):

∀ (x | r . ¬p) ≡ <st6> ¬¬∀ (x | r . ¬p)
≡ <ax25> ¬∃ (x | r . p)

Stelling 76: (a):

∃ (x | r . p) ≡ <ax25> ¬∀ (x | r . ¬p)
≡ <st65a> ¬∀ (x . ¬r ∨ ¬p)
≡ <st29a> ¬∀ (x . ¬(r ∧ p))
≡ <ax25> ∃ (x . r ∧ p)

Stelling 76: (b):

∃ (x | q ∧ r . p) ≡ <ax25> ¬∀ (x | q ∧ r . ¬p)
≡ <ax23> ¬∀ (x . (q ∧ r) => ¬p)
≡ <st42> ¬∀ (x . (q => (r => ¬p)))
≡ <st36> ¬∀ (x . (q => (¬r ∨ ¬p)))
≡ <st29a> ¬∀ (x . (q => (¬(r ∧ p)))
≡ <ax23> ¬∀ (x | q . ¬(r ∧ p))
≡ <ax25> ∃ (x | q . r ∧ p)

Stelling 77:

q ∧∃ (x | r . p) ≡ <st6> ¬¬(q ∧ ∃(x | r . p)
≡ <st29a> ¬(¬q ∨ ¬∃(x | r . p)
≡ <st75c> ¬(¬q ∨ ∀ (x | r . ¬p)
≡ <ax24> ¬∀ (x | r . ¬q ∨ ¬p)
≡ <st29a> ¬∀ (x | r . ¬(q ∧ p))
≡ <ax25> ∃ (x | r . q ∧ p)

Stelling 78:

p ∧∃ (x . r) ≡ <st20> p ∧∃ (x | r ∧ r)
≡ <st76a> p ∧∃ (x | r . r)
≡ <st77> ∃ (x | r . p ∧ r)
≡ <st76a> ∃ (x . r ∧ p ∧ r)
≡ <st20> ∃ (x . r ∧ p)
≡ <st76a> ∃ (x | r . p)

Stelling 79:

∃ (x . r) ≡ <ax25> ¬∀ (x . ¬r)
=> <st68> ∀ (x | r . ¬p ∧ ¬q) ≡ ¬p ∧ ∀ (x | r . ¬q)
≡ <st29b> ∀ (x | r . ¬(p ∨ q)) ≡ ¬p ∧ ∀ (x | r . ¬q)
≡ <st75c> ¬∃(x | r . p ∨ q) ≡ ¬p ∧ ¬∃(x | r . q)
≡ <st29b> ¬∃(x | r . p ∨ q) ≡ ¬(p ∨ ∃(x | r . q))
≡ <st5> ∃ (x | r . p ∨ q) ≡ p ∨∃ (x | r . q)

Stelling 80:

∃ (x | r . 0) ≡ <st76a> ∃ (x . r ∧ 0)
≡ <st18> ∃ (x . 0 ∧ r)
≡ <st76a> ∃ (x | 0 . r)
≡ <ax16b> 0

Stelling 81:

∃ (x | r . p) => ∃ (x | q ∨ r . p)
≡ <ax19b> ∃ (x | r . p) => ∃ (x | q . p) ∨ ∃ (x | r . p) (st53a)

Stelling 82:

∃ (x | r . p) => ∃ (x | r . p ∨ q)
≡ <ax18b> ∃ (x | r . p) => ∃ (x | r . p) ∨ ∃ (x | r . q) (st53a)

Stelling 83:

∀ (x | r . q => p) ≡ <st38> ∀ (x | r . ¬p => ¬q)
=> <st73> ∀ (x | r . ¬p) => ∀ (x | r . ¬q)
≡ <st75c> ¬∃ (x | r . p) => ¬∃ (x | r . q)
≡ <st38> ∃ (x | r . q) => ∃ (x | r . p)

Stelling 84:

p[x := e] => ∃ (x . p)
≡ <st38> ¬∃ (x . p) => ¬(p[x := e])
≡ <st75c> ∀ (x . ¬p) => ¬(p[x := e])
≡ <subst> ∀ (x . ¬p) => ¬(p)[x := e]
→ inst v st74

Stelling 85:

∃ (x | r . ∀ (y | q . p)) ≡ <st75c> ¬∀ (x | r . ¬∀ (y | q . p))
≡ <st74, st84>¬∃ (x | r . ¬∀ (y | q . p))
≡ <st74, st84>¬∃ (x | r . ¬∃ (y | q . p))
≡ <st75a> ∀ (y | q. ∃ (x | r . p))

Stelling 86:

∀ (x | r . p) ∨ ∀ (x | r . q) => ∀ (x | r. p ∨ q)
≡ <st25a> (∀ (x | r . p ∧ (p ∨ q)) ∨ ∀ (x | r . q ∧ (p ∨ q)) => ∀ (x | r. p ∨ q)
≡ <st55> (∀ (x | r . p ∧ (p ∨ q)) => ∀ (x | r . p ∨ q))
∧ (∀ (x | r . q ∧ (p ∨ q)) => ∀ (x | r. p ∨ q))
≡ <st72> 1 ∧ 1
≡ <st20> 1

Stelling 87:

∃ (x | r. p ∧ q) ≡ <st75b> ¬∀ (x | r. ¬(p ∧ q))
≡ <st29a> ¬∀ (x | r. ¬p ∨ ¬q)
=> <st86> ¬(∀ (x | r. ¬p) ∨ ∀ (x | r. ¬q))
≡ <st29b> ¬∀ (x | r. ¬p) ∧ ¬∀ (x | r. ¬q)
≡ <st75c> ∃ (x | r. p) ∧ ∃ (x | r. q)