oef 1.1) 1

$$\begin{cases} x^2y' + 2xy - 1 = 0 \\ y(x_0) = y_0 \quad (x_0 > 0,\ y_0 \in \mathbb{R}) \end{cases} \rightarrow \text{beginvoorw.}$$

$$x^2y' + 2xy - 1 = 0$$

$$\Leftrightarrow \quad y' + \frac{2}{x}y = \frac{1}{x^2}$$

$$\rightsquigarrow \quad y = e^{-\int \frac{2}{x}dx}\left(C + \int \frac{1}{x^2} \cdot e^{\int \frac{2}{x}dx} dx\right)$$

$$\Leftrightarrow \quad y = e^{-2\ln x}\left(C + \int \frac{e^{\ln x^2}}{x^2} dx\right)$$

$$\Leftrightarrow \quad y = e^{-2\ln x}\left(C + \int dx\right)$$

$$\Leftrightarrow \quad y = \frac{1}{x^2}\left(C + x\right)$$

$$\Leftrightarrow \quad y = \frac{C}{x^2} + \frac{1}{x}$$

$$\rightsquigarrow \quad y(x_0) = \frac{C}{x_0^2} + \frac{1}{x_0} = y_0 \Leftrightarrow C = \left(y_0 - \frac{1}{x_0}\right)x_0^2$$

$$\Leftrightarrow C = y_0x_0^2 - x_0$$

$$\rightsquigarrow \text{OPLOSSING: } y(x) = \frac{y_0x_0^2 - x_0}{x^2} + \frac{1}{x}$$

oef 12 | 3 $\begin{cases} y'' + y' - 2y = 0 \\ y(0) = 1,\ y'(0) = 0 \end{cases}$

$\rightsquigarrow$ karakt: $\lambda^2 + \lambda - 2 = 0$

$D = 9 \to \lambda_{1,2} = \dfrac{-1 \pm 3}{2} \begin{matrix} \nearrow 1 \\ \searrow -2 \end{matrix}$

$\rightsquigarrow$ oplossingen: $\varphi_1(x) = e^x$ en $\varphi_2(x) = e^{-2x}$

$\hookrightarrow y(x) = c_1 e^x + c_2 e^{-2x}$

$\rightsquigarrow$ c_1, c_2 bepalen:

$\Rightarrow \begin{cases} y(0) = c_1 + c_2 = 1 \\ y'(0) = c_1 - 2c_2 = 0 \end{cases} \Leftrightarrow \begin{cases} 3c_1 = 1 \\ c_1 = 2c_2 \end{cases}$

$\Leftrightarrow \begin{cases} c_1 = \frac{1}{3} \\ c_2 = \frac{2}{3} \end{cases}$

$\rightsquigarrow$ OPLOSSING: $y(x) = \dfrac{e^x}{3} + \dfrac{2e^{-2x}}{3}$

oef 1,4 ② $y'' + y = 3 + 6e^{-2x}$

homogene:

$y'' + y = 0 \rightarrow$ karakt vgl: $\lambda^2 + 1 = 0$
$\Leftrightarrow \lambda = \pm i$

$\rightarrow$ oplossingen $\varphi_1(x) = e^{0x}\cos(x) = \cos x$
$\varphi_2(x) = e^{0x}\sin(x) = \sin x$

$\rightarrow y_{(h)}(x) = c_1\cos x + c_2 \sin x$

particulier: superpositie:

$y'' + y = 3 \rightarrow$ stel $y = 3$

$y'' + y = 6e^{-2x}$

$\hookrightarrow 6e^{-2x} = \cancel{e^{ax}P(x)}$ ~~en~~ $\cancel{y(x) = e^{-2x} \cdot Q(x)}$

$a = -2$

$\hookrightarrow \cancel{P(x) = Q''(x)}$

$\Rightarrow \cancel{Q''(x) = 6 \Leftrightarrow \int 6dx = Q'(x) + c}$

$\Leftrightarrow \cancel{\int 6x\,dx = Q(x) + c}$

$\Leftrightarrow \cancel{3x + \underbrace{c}_{\text{stel } 0} = Q(x)}$

$6e^{-2x} = e^{-2x}(6 \cdot \cos(0) + 0) \rightarrow$ $a = -2$, $b = 0$, $s = 0$, $c = 6$

$= e^{-2x} \cdot 6$

$a + bi = -2 \rightarrow$ Geen wortel

$\rightsquigarrow y(x) = e^{-2x} \cdot c_0$

$y'(x) = -2e^{-2x}c_0$

$y''(x) = 4e^{-2x}c_0$

$4e^{-2x}c_0 + e^{-2x}c_0 = 6e^{-2x}$

$\Leftrightarrow 5\cancel{e^{-2x}}c_0 = 6\cancel{e^{-2x}}$

$\Leftrightarrow c_0 = \frac{6}{5} \rightarrow y_{(p)} = 3 + \frac{5}{6}e^{-2x}$

OPLOSSING: $y(x) = 3 + \frac{5}{6}e^{-2x} + c_1\cos x + c_2 \sin x$

④ $y'' - 2y' + y = (x+1)e^{2x} + e^x - 1$

homogene : $y'' - 2y' + y = 0$

$\rightsquigarrow$ karakt: $\lambda^2 - 2\lambda + 1 = 0 \Leftrightarrow (\lambda - \frac{2}{2})^2 = 0$

$D = 4 - 4 = 0$ $\qquad \frac{2}{2} = 1$

$\rightsquigarrow$ oplossingen : $y_{(h)}(x) = c_1 e^x + c_2 x e^x$ $\quad 2x$

partikulier : Superpositie :

$y'' - 2y' + y = (x+1)e^{2x}$ (1)

$y'' - 2y' + y = e^x$ (2)

$y'' - 2y' + y = -1$ (3) $\longrightarrow y = -1$

(1) $y'' - 2y' + y = (x+1)e^{2x}$

$(x+1)e^{2x} = e^{ax}(C(x)\cos(bx) + S(x)\sin(bx))$

$\hookrightarrow a = 2,\ b = 0,\ C(x) = (x+1)$

$\hookrightarrow a + bi = 2 \rightarrow$ geen wortel

$\rightsquigarrow y(x) = e^{2x}(C_0(x))$: $e^{2x} C_0'(x)$

$\rightsquigarrow y'(x) = 2e^{2x} C_0(x) + e^{2x} C_0'(x)$

$\rightsquigarrow y''(x) = 4e^{2x} C_0(x) + 2e^{2x} C_0'(x) + e^{2x} \cancel{C_0''(x)}$
$+ 2e^{2x} C_0'(x)$ $\qquad \hookrightarrow (cte)'$

$\rightarrow \cancel{e}^{2x} C_0(x) - \cancel{4e^{2x} C_0(x)} - 2e^{2x} \cancel{C_0'(x)}$
$+ \cancel{4e^{2x} C_0(x)} + 2e^{2x} \cancel{C_0'(x)}$
$+ 2\cancel{e}^{2x} C_0'(x) = (x+1) \cancel{e}^{2x}$

$\Leftrightarrow \cancel{2} C_0'(x) + \frac{C_0(x)}{2} = \frac{x+1}{2}$

$\Leftrightarrow C_0(x) = e^{\frac{x}{2}}(C + \int \frac{x+1}{2} \cdot e^{\frac{x}{2}} dx)$

$C_0(x) = 0$

(2) $e^x = e^{ax}$ met $a = 1$, $b = 0$,
$\hookrightarrow a + bi = 1 \rightarrow$ dubbel wortel !

$e^x = e^x \cdot P(x)$ met $P(x) = 1$

$y = e^x \cdot Q(x)$ $\qquad Q'(x) = x + C_3$

$Q''(x) = P(x) \Leftrightarrow Q(x) = \frac{x^2}{2} + x C_3 + C_4$

$\rightarrow$ OPLOSSING:

$y(x) = \frac{x^2}{2} + x C_3 + C_4 + (-1) + e^{\frac{x}{2}}(C + \int \frac{x+1}{2} e^{\frac{x}{2}} dx)$
$+ c_1 e^x + c_2 x e^x$

oef 1.4 6 $y'' - 2y' + y = \sin x + \sinh x = \sin x + \frac{e^x}{2} - \frac{e^{-x}}{2}$

→ homogeen: $y'' - 2y' + y = 0$

karakt: $x^2 - 2x + 1 = 0$

$D = 4 - 4 = 0 \Rightarrow x_{1,2} = \frac{+2}{2} = 1$

$\rightsquigarrow \varphi_1(x) = e^x \quad , \quad \varphi_2(x) = xe^x$

$\rightsquigarrow y = c_1 e^x + c_2 x e^x$

→ particulier

superpositie: $y'' - 2y' + y = \sin x \quad (1)$

$y'' - 2y' + y = \frac{e^x}{2} \quad (2)$

$y'' - 2y' + y = -\frac{e^{-x}}{2} \quad (3)$

(1): $\sin x = e^{ax}(C(x)\cos(bx) + S(x)\sin(bx))$

$a = 0$, $b = 1$ | $\left.\begin{array}{l} C_0(x) = 0 \\ S_0(x) = 1 \end{array}\right\}$ hoogste graad 0

$a + bi = i \rightsquigarrow$ geen wortel

$y_{p_1}(x) = e^{ax}(C_0(x)\cos(bx) + S_0(x)\sin(bx))$

$= \underbrace{C_0(x)}_{c^{te}}\cos x + \underbrace{S_0(x)}_{c^{te}}\sin x$

$y'_{p_1}(x) = -C_0 \sin x + S_0 \cos x$

$y''_{p_1}(x) = -C_0 \cos x - S_0 \sin x$

$\rightsquigarrow \cancel{-C_0\cos x} - \cancel{S_0 \sin x} + 2C_0 \sin x - 2S_0 \cos x + \cancel{C_0 \cos x} + \cancel{S_0 \sin x} = \sin x$

$\Leftrightarrow C_0 \sin x - S_0 \cos x = \frac{\sin x}{2}$

$\Rightarrow \begin{cases} C_0 = \frac{1}{2} \\ S_0 = 0 \end{cases}$

$\Rightarrow y_{p_1}(x) = \frac{1}{2}\cos x$

oef 1 4.) ⑦ $\begin{cases} y'' - 2y' = e^{2x} + x^2 - 1 \\ y(0) = \frac{1}{8}, \; y'(0) = 1 \end{cases}$

homogeen: $y'' - 2y' = 0$

$\rightsquigarrow$ karakt: $\lambda^2 - 2\lambda = 0$

$\Leftrightarrow \lambda(\lambda - 2) = 0$

$\varphi_1(x) = 1 \quad , \quad \varphi_2(x) = e^{2x}$

$y_h(x) = C_1 + C_2 e^{2x}$

$\Rightarrow \begin{cases} y(0) = C_1 + C_2 = \frac{1}{8} \Leftrightarrow C_1 = -\frac{3}{8} \\ 2C_2 = 1 \Leftrightarrow C_2 = \frac{1}{2} \end{cases}$

particulier

$y'' - 2y' = e^{2x} \quad (1) \quad a = 2, b = 0, C(x) = 0$

$y'' - 2y' = x^2 \; (2)$

$y'' - 2y' = 1 \quad \to y' = 1 \Leftrightarrow y = x + C_3 - x$

(1) $(y')' - 2y' = e^{2x}$

$a = -2$

$y' = e^{+2x}\left(C + \int 1 \, dx\right) = e^{2x} C + x + C_4$

$\Leftrightarrow y = \frac{e^{2x}}{2} C + \frac{x^2}{2} + C_5$

(2) $(y')' - 2y' = x^2$

$y' = e^{2x}\left(C + \int x^2 \cdot e^{-2x} \, dx\right)$

(2) $y(x) = \frac{e^x}{2} = e^x \cdot Q(x) \qquad (a = 1, b = 0)$

$\rightsquigarrow \frac{e^x}{2} = P(x) e^x \qquad Q = \iint P(x)\,dx$

$= \iint \frac{1}{2}\,dx$

$= \int (\frac{x}{2} + C)\,dx$

$= \frac{x^2}{4} + Cx + C'$

$\rightsquigarrow \frac{e^x}{2} = (\frac{x^2}{4} + Cx + C')e^x$

$\boxed{y_{P_2}(x) = \frac{x^2}{4} e^x} + \underbrace{Cxe^x + C'e^x}_{\text{homogene}}$

(3) $y(x) = -\frac{e^x}{2} = Ce^{-x} \qquad (a = -1, b = 0)$

$\rightsquigarrow y'(x) = -Ce^{-x}$

$\rightsquigarrow y''(x) = Ce^{-x}$

$\Rightarrow e^{-x}(C + 2C + C) = \frac{e^{-x}}{2}$

$\Rightarrow C = \frac{1}{8} \Rightarrow y_{P_3}(x) = -\frac{1}{8} e^{-x}$

$\overset{!!}{\smile} \quad (1) + (2) + (3)$

OPLOSSING :

$y(x) = c_1 e^x + c_2 x e^x + \frac{1}{2}\cos x + \frac{x^2}{4} e^x - \frac{1}{8} e^{-x}$

met $c_1, c_2 \in \mathbb{C}$

oef 1.8 ①

<u>inleiding</u>: verlagen van de orde

$$y'' + py' + qy = R(x)$$

Stel $y = \varphi z$ (gokken dat dat zo is)

$\Rightarrow \varphi(x) z'' + (2\varphi'(x) + p\varphi(x)) z' + \underbrace{(\varphi''(x) + p\varphi'(x) + q\varphi(x))}_{\substack{* \\ \text{Stel } 0}} z = R(x)$

1e orde vgl in z': O.K.

$\Rightarrow z = \ldots$

$\Rightarrow y = \varphi z$

$*$ Neem φ opl. hom. vgl.

<u>oefening</u>

$$y'' + y = \tan x \quad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

<u>homogeen</u>: $\lambda^2 + 1 = 0 \iff \lambda = \pm i$

$\rightsquigarrow y_h(x) = c_1 \sin x + c_2 \cos x$

<u>particulier</u>:

$\rightsquigarrow$ verlagen van orde: opl homogene!

Stel $y_p(x) = \varphi z$, $\varphi = \cos x$

$\rightsquigarrow z'' \cos x + (-2\sin x) z' = \tan x = \frac{\sin x}{\cos x}$

1e orde in z' $\leftarrow$ $\iff (z')' \cos x + (-2\sin x) z' = \frac{\sin x}{\cos x} \quad \Big| \cdot \frac{1}{\cos^2 x}$

$\iff (z')' + \left(-2\frac{\sin x}{\cos x}\right) z' = \frac{\sin x}{\cos^2 x}$

$e^{-\int a}(c + \int R e^{\int a} dx)$

$c = 0$ $\rightsquigarrow$ $\int \frac{1}{\cos x} = \frac{\ln}{2} |\tan x + \sec x| (+C)$

$\int \frac{\sin x}{\cos x} = -\ln |\cos x| (+C)$

$\rightsquigarrow z' = e^{-2\ln(\cos x)} \left(\int \frac{\sin x}{\cancel{\cos^2 x}} e^{\cancel{2\ln}\cos x} dx\right)$

$= -\frac{1}{\cos^2 x} \cdot \cos x = -\frac{1}{\cos x}$

$\rightsquigarrow z = -\int \frac{1}{\cos x} dx = -\frac{1}{2} \ln |\tan x + \sec x|$

$\rightsquigarrow y_p(x) = \cos x \cdot \left(-\frac{1}{2} \ln |\tan x + \sec x|\right)$

$\Rightarrow$ OPLOSSING: $y(x) = c_1 \sin x + c_2 \cos x + \cos x \left(-\frac{1}{2} \ln |\tan x + \sec x|\right)$

③ $y'' - 2y' + y = \frac{e^x}{(x-1)^2}$

homogene : $y'' - 2y' + y = 0$

$\rightsquigarrow$ karakt. : $\lambda^2 - 2\lambda + 1 = 0$

$\lambda = 1$

$\rightsquigarrow y_h(x) = c_1 e^x + c_2 x e^x$

particulier : verlagen vd orde :

stel $y = \varphi z$, $\varphi = e^x$

$\rightsquigarrow (e^x z)'' - 2(e^x z)' + e^x z$

$= (e^x \cdot z + e^x \cdot z')' - 2(e^x z + e^x z') + e^x z$

$= \cancel{e^x z} + e^x z' + e^x z' + e^x z'' \cancel{- 2e^x z} - e^x z' \cdot 2 + \cancel{e^x z}$

$= \cancel{e^x}(z')' = \frac{\cancel{e^x}}{(x-1)^2}$: eerste orde in z'

$\rightsquigarrow a = 0 \Rightarrow e^{-\int a} = e^0 = 1$

$\rightsquigarrow z' = 1 \cdot \left(c + \int \frac{1}{(x-1)^2} \cdot 1\, dx\right)$

$\Leftrightarrow z' = c + \int \frac{1}{(x-1)^2} dx$

$\Leftrightarrow z' = c + \left(-\frac{1}{x-1} + c\right)$

$\Rightarrow z = -\ln(x-1) + \underset{0}{c}$ want we zoeken slechts 1 specifieke opl.

x is voldoende groot

$\rightsquigarrow$ OPLOSSING :

$y(x) = c_1 e^x + c_2 x e^x - \ln(x-1) e^x$

oefening 1)

① Fourierontwikkeling van $f(x) = x$ tov $[-\pi, \pi]$

→ oneven functie. $a_n = 0 \quad (n = 0, 1, 2, \dots)$

$$b_n = \frac{2}{\pi}\int_0^{\pi} x \sin(nx)\,dx$$

$$\overset{PI}{=} \frac{-2}{n\pi}\left([x\cos(nx)]_0^{\pi} - \int_0^{\pi}\cos(nx)\,dx\right)$$

$$= \frac{-2}{n\pi}\left((-1)^n\pi - \left[\frac{\sin(nx)}{n}\right]_0^{\pi}\right)$$

$$= -\frac{2}{n\pi}(-1)^n\pi = \frac{(-1)^{n+1}2}{n}$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{+\infty}\left(a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right)$$

$$= \sum_{n=1}^{+\infty}\left(\frac{(-1)^{n+1}2}{n}\sin nx\right)$$

$$= 2\sin x - \frac{2}{2}\sin(2x) + \frac{2}{3}\sin(3x) - \frac{1}{2}\sin(4x) + \dots$$

$\forall x \in [-\pi, \pi]$

---

② $f(x) = |x|$

→ even. $b_n = 0$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} = \pi$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x\cos(nx)\,dx \overset{PI}{=} \frac{2}{n\pi}\left[[x\sin(nx)]_{x=0}^{x=\pi} - \int_0^{\pi}\sin(nx)\,dx\right]$$

!!

$$= \frac{2}{n\pi}\cdot\left[-\frac{\cos(nx)}{n}\right]_0^{\pi} = \frac{-2(-1)}{n^2\pi} \overset{\neq\frac{2}{n\pi}}{=} \frac{(-1)^{n+1}2}{n^2\pi} \neq \frac{2}{n^2\pi}$$

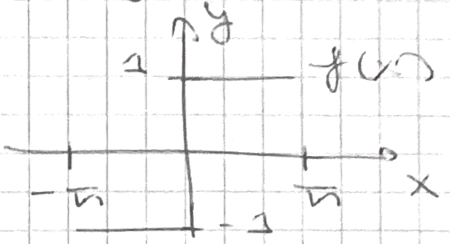
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{+\infty}\left(a_n\cos(nx) + b_n\sin(nx)\right)$$

$$= \frac{\pi}{2} + \frac{4}{\pi}\cos x - \cancel{\frac{2}{4\pi}\cos(2x)} + \frac{4}{9\pi}\cos 3x - \cancel{\frac{2}{16\pi}\cos(4x)} + \dots$$

$\forall x \in [-\pi, \pi]$

③

$$f(x) = \begin{cases} -1 & \forall x \in \left](2k-1)\pi, 2k\pi\right[\\ 1 & \forall x \in \left]2k\pi, (2k+1)\pi\right[\end{cases} \quad \text{neem } k=0$$



$a_n = 0 \ (n=0,1,2,\dots) \leftarrow$ oneven $\leftarrow$

$$b_n = \frac{2}{\pi}\int_0^{\pi} \sin(nx)\,dx$$

$$= -\frac{2}{\pi}\left[\frac{\cos(nx)}{n}\right]_0^{\pi} = -\frac{2}{\pi}\frac{(-1)^n}{n} + \frac{2}{\pi n}$$

$$= \frac{((-1)^{n+1}+1)2}{\pi n}$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{+\infty}\left(a_n\cos(nx) + b_n\sin(nx)\right)$$

$$= \sum_{n=1}^{+\infty}\left(\frac{((-1)^{n+1}+1)2}{n\pi}\sin(nx)\right)$$

$$= \frac{4}{\pi}\sin x + \frac{4}{3\pi}\sin(3x) + \frac{4}{5\pi}\sin(5x) + \dots$$

$\forall x \in \left]-\pi, 0\right[\cup \left]0, \pi\right[$.

---

④ $f(x) = x^2 \rightarrow$ even : $b_n = 0$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x^2\,dx = \frac{2}{\pi}\frac{\pi^3}{3} = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2\cos(nx)\,dx$$

$$\overset{PI}{=} \frac{2}{\pi n}\left(\left[x^2\sin(nx)\right]_0^{\pi} - \int_0^{\pi} 2x\sin(nx)\,dx\right)$$

$$= +\frac{4}{\pi n^2}\left(\left[x\cos(nx)\right]_0^{\pi} - \int_0^{\pi}\cos(nx)\,dx\right)$$

$$= \frac{4}{\pi n^2}\pi\cdot(-1)^n - \frac{4}{\pi n^3}\left[\sin(nx)\right]_0^{\pi}$$

$$= \frac{(-1)^n 4}{n^2}$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{+\infty}\left(a_n\cos(nx) + b_n\sin(nx)\right)$$

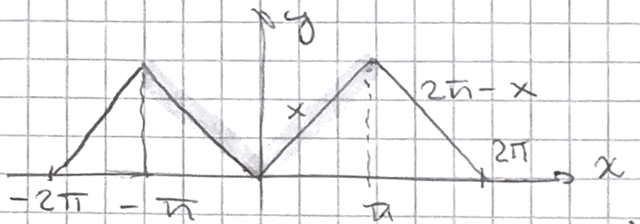
$$= \frac{\pi^2}{3} - 4\cos(x) + \cos(2x) - \frac{4}{9}\cos(3x) + \dots$$

$\forall x \in [-\pi, \pi]$

oefening 2:

$$f(x) = \begin{cases} x & \forall x \in [0, \pi] \\ 2\pi - x & \forall x \in [\pi, 2\pi] \end{cases}$$

we willen $f(x)$ ontwikkelen in een reeks van cosinussen → even → $b_n = 0$



we breiden f periodiek uit

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2\pi}{2} = \pi$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x\cos(nx)\,dx$$

$$= -\frac{2}{\pi n}\left([x\sin(nx)]_0^{\pi} - \int_0^{\pi}\sin(nx)\,dx \right)$$

$$= \frac{+2}{\pi n}\left[-\frac{\cos(nx)}{n}\right]_0^{\pi}$$

$$= \frac{2}{\pi n^2}\left(-\cos(\pi x) + 1\right)$$

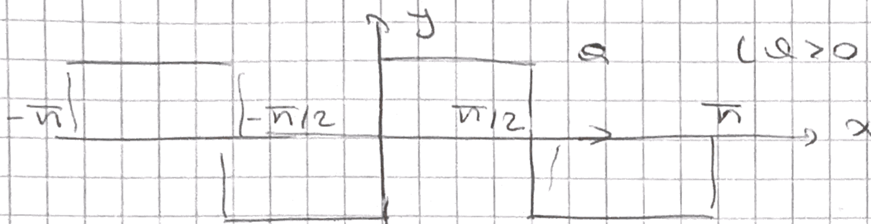
$$= \frac{2((-1)^{n+1} + 1)}{\pi n^2}$$

$$f(x) \sim \frac{\pi}{2} + \frac{4}{\pi}\cos x + \frac{4}{9\pi}\cos 3x + \dots \quad \forall x \in [0, 2\pi]$$

oefening 3:

$$f(x) = \begin{cases} a & \forall x \in [0, \pi/2] \\ -a & \forall x \in [\pi/2, \pi] \end{cases} \quad (a \text{ is constant})$$

in een reeks van sinussen → oneven



(a > 0 & in schets)

$a_n = 0 \quad (n = 0, 1, \ldots)$

$$b_n = \frac{2}{\pi} \cdot 2 \int_0^{\pi/2} a \sin\left(\frac{n x \pi}{\pi/2}\right) dx$$

$$= \frac{4}{\pi} \int_0^{\pi/2} a \sin(2nx)\, dx$$

$$= -\frac{4a}{\pi} \cdot \frac{1}{2n} \left[\cos(2nx)\right]_0^{\pi/2}$$

$$= -\frac{2a}{n\pi} \left(\cos(n\pi) - \cos 0\right)$$

$$= -\frac{2a}{n\pi}\left((-1)^n - 1\right) = \frac{2a((-1)^{n+1} + 1)}{n\pi} = \begin{cases} 0 & \text{voor } n \text{ even} \\ \frac{4a}{n\pi} & \text{voor } n \text{ oneven} \end{cases}$$

$$f(x) \sim \sum_{n=1}^{+\infty} \frac{2a((-1)^{n+1} + 1)}{n\pi} \sin(2nx)$$

$$= \frac{4a}{\pi} \sin(2x) + \frac{4a}{3\pi} \sin(6x) + \frac{4a}{5\pi} \sin(10x) + \ldots$$

$\forall x \in \left]0, \frac{\pi}{2}\right[\cup \left]\frac{\pi}{2}, \pi\right[$

f van $[-\pi, \pi]$

$$f \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx) \quad *$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(n) \cos(nx)\,dx$$

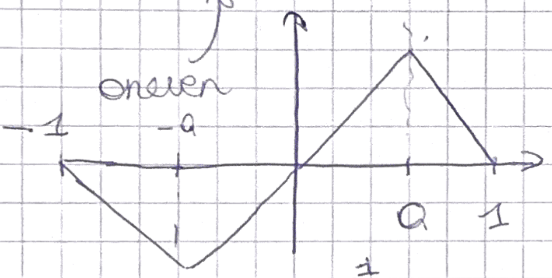
$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\,dx$$

Als f stuksgewijs C^1, dan $f^{\pi,v}(x) = *$

oef 4)

$\sum \sin$ $\quad = f(x) = \begin{cases} (1-a)x & \forall x \in [0,a] \\ a(1-x) & \forall x \in [a,1] \end{cases}$ met $0 < a < 1$ constant

oneven



$$\cancel{a_0 = \frac{1}{-1} \int_{-1}^{1} f(x)\,dx = -2 \int_0^a (1-a)x\,dx - 2 \int_a^1 a(1-x)\,dx}$$

$$= -2 \left[\left[\frac{1-a}{2} x^2 \right]_0^a + \left[ax - \frac{a x^2}{2} \right]_a^1 \right]$$

$$= -2 \left[\frac{a - a^2}{2} + a - \frac{a}{2} + \left(-\left(a^2 - \frac{a^3}{2} \right) \right) \right]$$

$$= -2 \left[\cancel{\frac{a}{2}} - \frac{a^2}{2} + a - \cancel{\frac{a}{2}} - a^2 + \frac{a^3}{2} \right]$$

$$= a^2 - 2a + a^2 - a^3$$

$$= -2a + 2a^2 - a^3$$

$a_n = 0 \quad \forall n \in \mathbb{N}$

$$b_n = 2 \int_0^a (1-a)x \sin(n\pi x)\,dx + 2 \int_a^1 a(1-x) \sin(n\pi x)\,dx$$

PI/S ...
$$= 2(1-a)a \frac{\cos n\pi a}{n\pi} + 2(1-a) \frac{\sin a n\pi}{(n\pi)^2} + 2(1-a)a \frac{\cos n\pi a}{n\pi} + \frac{2a \sin n\pi a}{(n\pi)^2}$$

oefening 4.3.23.

1) $A = \begin{pmatrix} 7 & 6 & 1 \\ 0 & 4 & 4 \\ -1 & -2 & 1 \end{pmatrix}$

karakteristieke veelterm:

$\det(xI - A) = \left| \begin{pmatrix} x-7 & -6 & -1 \\ 0 & x-4 & -4 \\ 1 & 2 & x-1 \end{pmatrix} \right|$

$= (x-1)(x-4)(x-7) + 24 - (-x+4 - 8x + 56)$

$= (x^2 - 4x - x + 4)(x-7) + 24 - (-9x + 60)$

$= (x^2 - 5x + 4)(x-7) + 24 + 9x - 60$

$= x^3 - 7x^2 - 5x^2 + 35x + 4x - 28 + 24 - 60 + 9x$

$= x^3 - 12x^2 + 39x + 9x - 4 - 60$

$= x^3 - 12x^2 + 48x - 64 = 0$

$x = 4: \quad 4^3 - 12 \cdot 4^2 + 48 \cdot 4 - 64 = 0$

	1	-12	48	-64
4		4	-32	64
	1	-8	16	0

$x^2 - 8x + 16 = (x-4)^2$

$\rightarrow x^3 - 12x^2 + 48x - 64 = (x-4)^3$

$\left(\begin{array}{ccc|c} -3 & -6 & -1 & 0 \\ 0 & 0 & -4 & 0 \\ 1 & 2 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 6 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow$

$\sim \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \begin{cases} x + 2y = 0 \\ z = 0 \\ y = r \end{cases} \rightarrow \begin{cases} x = -2r \\ z = 0 \\ y = r \end{cases}$

$\rightarrow \langle -2, 1, 0 \rangle$ algebraische multiplicitеit is kleiner dan meetkundig:

$\left(\begin{array}{ccc|c} -3 & -6 & -1 & -2 \\ 0 & 0 & -4 & 1 \\ 1 & 2 & 3 & 0 \end{array}\right) \overset{rref}{\sim} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 3/4 \\ 0 & 0 & 1 & -1/4 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\rightarrow \begin{cases} x = 3/4 - 2y \\ y = r \\ z = -1/4 \end{cases} \rightarrow (3/4, 0, -1/4) + \langle -2, 1, 0 \rangle$

$\left(\begin{array}{ccc|c} -3 & -6 & -1 & 3/4 \\ 0 & 0 & -4 & 0 \\ 1 & 2 & 3 & -1/4 \end{array}\right) \overset{rref}{\sim} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1/4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \begin{cases} x = -1/4 - 2y \\ z = 0 \\ y = r \end{cases}$

$\rightarrow (-1/4, 0, 0) + \langle -2, 1, 0 \rangle$

basis: $\left\{ e^{4t} c_0 ,\ e^{4t}(c_0 t + c_1),\ e^{4t}\left(c_0 \frac{t^2}{2} + c_1 t + c_2\right) \right\}$

met

$$c_0 = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix},\quad c_1 = \begin{pmatrix} 3/4 \\ 0 \\ -1/4 \end{pmatrix},\quad c_2 = \begin{pmatrix} -1/4 \\ 0 \\ 0 \end{pmatrix}$$

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2)
$$A = \begin{pmatrix} 2 & -1 & 5 \\ 0 & -2 & 16 \\ 0 & -1 & 6 \end{pmatrix}$$

karakteristieke vergelijking:

$$\det(xI - A) = \begin{vmatrix} x-2 & 1 & -5 \\ 0 & x+2 & -16 \\ 0 & 1 & x-6 \end{vmatrix}$$

$= (x^2-4)(x-6) + 16(x-2) = x^3 - 6x^2 - 4x + 24 + 16x - 32$

$= x^3 - 6x^2 + 12x - 8 = 0$

nulwaarden: $x = 2$

$$\begin{array}{c|cccc} & 1 & -6 & 12 & -8 \\ 2 & & 2 & -8 & 8 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$x^2 - 4x + 4 = 0 \Leftrightarrow (x-2)^2 = 0$

dus we hebben 3x $x = 2$ als eigenwaarde:

$$\rightarrow \left(\begin{array}{ccc|c} 0 & 1 & -5 & 0 \\ 0 & 4 & -16 & 0 \\ 0 & 1 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 1 & -5 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 1 & -5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 0 & 1 & -5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \begin{cases} x = a \in \mathbb{R} \\ y = 0 \\ z = 0 \end{cases}$$

algebraische multipliciteit $\neq$ meetkundige

$$w_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\rightarrow \left(\begin{array}{ccc|c} 0 & 1 & -5 & 1 \\ 0 & 1 & -4 & 0 \\ 0 & 1 & -4 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 1 & -5 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\rightarrow \begin{cases} x = a \in \mathbb{R} \\ y = -4 \\ z = -1 \end{cases} \rightarrow \underbrace{\begin{pmatrix} 0 \\ -4 \\ -1 \end{pmatrix}}_{w_1} + a \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\rightarrow \left(\begin{array}{ccc|c} 0 & 1 & -5 & 0 \\ 0 & 4 & -16 & -4 \\ 0 & 1 & -4 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 1 & -5 & 0 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\rightarrow \begin{cases} x = a \in \mathbb{R} \\ y = -5 \\ z = -1 \end{cases} \rightarrow \begin{pmatrix} 0 \\ -5 \\ -1 \end{pmatrix} + s\begin{pmatrix} 0 \\ -4 \\ -1 \end{pmatrix} + \frac{a^2}{2}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

basis: $\left\{ e^{2a}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},\ e^{2a}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} a + \begin{pmatrix} 0 \\ -4 \\ -1 \end{pmatrix}\right),\ e^{2a}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\frac{a^2}{2} + \begin{pmatrix} 0 \\ -4 \\ -1 \end{pmatrix} a + \begin{pmatrix} 0 \\ -5 \\ -1 \end{pmatrix}\right) \right\}$