

# Propositie rekenen Stellingen

- Stelling 1 cursus p. 42
- Stelling 2 " p. 44
- Stelling 3 " " "
- Stelling 4 " " 47
- Stelling 5 " " "
- Stelling 6  $\neg\neg x \equiv x$

$\langle \text{nr 5} \rangle \quad \neg (y \equiv x \equiv \neg y) \equiv x$ $\langle \text{nr 5} \rangle \quad \neg y \equiv x \equiv \neg y \equiv x$ $\langle \text{ax 2, } x, y := \neg y, y \rangle$	$\langle \text{nr 5, } \neg x \equiv y \equiv x \equiv \neg y$ met $x, y := \neg x, x \rangle \quad \neg x \equiv \neg x$
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Stelling 7

$$\neg 0 \equiv 1$$

$$\langle \text{ax 4} \rangle \quad \neg\neg 1 \equiv 1$$

$$\langle \text{nr 6, } x := 1 \rangle \quad \neg\neg x \equiv x$$

Stelling 8

$$(x \neq y) \equiv \neg x \equiv y$$

$$(x \neq y) \equiv \langle \text{ax 6} \rangle \quad \neg(x \equiv y)$$

$$\equiv \langle \text{ax 5} \rangle \quad \neg \forall \equiv y$$

Stelling 9 cursus p. 48

Stelling 10 " " "

Stelling 11

$$((x \neq y) \neq z) \equiv (x \neq (y \neq z))$$

$$((x \neq y) \neq z) \equiv \langle \text{nr 10, nr 8, } x, y := \neg(x \neq y), z \rangle \quad (z \neq (x \neq y))$$

$$\equiv \langle \text{nr 10} \rangle \quad (z \neq$$

$$((x \neq y) \neq z) \equiv (x \neq (y \neq z))$$

$$\langle \text{nr 8} \rangle \quad ((\neg x \equiv y) \neq z) \equiv (x \neq (\neg y \equiv z))$$

$$\langle \text{nr 10, nr 8} \rangle \quad ((\neg\neg x \equiv y) \equiv z) \equiv ((\neg\neg y \equiv z) \equiv x)$$

$$\langle \text{nr 6} \rangle \quad ((x \equiv y) \equiv z) \equiv ((y \equiv z) \equiv x)$$

$\leftarrow \text{sym v.} \equiv (\text{ax 2})$

$$\langle \text{nr ax 1} \rangle$$

Stelling 12 ①

$$((x \neq y) \equiv z) \equiv (x \neq (y \equiv z))$$

$$\equiv \langle \text{nr 8} \rangle \quad ((\neg x \equiv y) \equiv z) \equiv ((\neg y \equiv z \equiv x))$$

$$\equiv \langle \text{nr 10} \rangle \quad \neg x \equiv y \equiv z \equiv \neg y \equiv z \equiv x$$

$$\equiv \langle \text{nr 5} \rangle \quad x \equiv \neg y \equiv z \equiv \neg y \equiv z \equiv x$$

$\rightarrow \text{nr 3}$

③  $(x \neq y) \equiv z \equiv (x \neq (y \equiv z))$   
 $(\neg(x \equiv y) \equiv z) \equiv (\neg(x \equiv y \equiv z)) \rightarrow$  cont. n. 3

stelling 13

$$\begin{aligned} x \neq y \equiv z &\equiv x \equiv y \neq z \\ x \neq y \equiv z &\equiv \langle \text{nt } 8 \rangle \neg(x \equiv y \equiv z) \\ &\equiv \langle \text{ax } 9 \rangle x \equiv 0 \equiv y \equiv \neg z \equiv 0 \\ &\equiv \langle \text{nt } 4 \text{ y} := 0 \rangle x \equiv y \equiv \neg z \equiv 1 \\ &\equiv \langle \text{nt } 4 \text{ z} := 1 \rangle x \equiv y \equiv \neg z \\ &\equiv \langle \text{nt } 8 \rangle x \equiv y \neq z \end{aligned}$$

stelling 14

currus p. 43

stelling 15

$$\begin{aligned} x \vee 0 &\equiv x \\ x \vee 0 &\equiv \langle \text{ax } 9 \rangle x \vee (\neg x \equiv x) \\ &\equiv \langle \text{ax } 10 \rangle (x \vee \neg x) \equiv (x \vee x) \\ &\equiv \langle \text{ax } 11, 5 \rangle x \\ &\equiv \langle \text{ax } 3 \rangle x \end{aligned}$$

stelling 16

$$\begin{aligned} x \vee (y \vee z) &\equiv (x \vee y) \vee (x \vee z) \\ (x \vee y) \vee (x \vee z) &\equiv \langle \text{ax } 7 \rangle (y \vee x) \vee (x \vee z) \\ &\equiv \langle \text{ax } 8 \rangle x \vee (y \vee (x \vee z)) \\ &\equiv \langle \text{ax } 7 \rangle (y \vee ((x \vee x) \vee z)) \\ &\equiv \langle \text{ax } 9 \rangle (y \vee (x \vee z)) \\ &\equiv \langle \text{ax } 8 \rangle (y \vee x) \vee z \\ &\equiv \langle \text{ax } 7, 9 \rangle (x \vee (y \vee z)) \end{aligned}$$

stelling 17

$$\begin{aligned} (x \vee y) &\equiv (x \vee \neg y) \equiv x \\ (x \vee y) \equiv (x \vee \neg y) &\equiv \langle \text{ax } 10 \rangle x \vee (y \equiv \neg y) \\ &\equiv \langle \text{nt } 5 \rangle x \vee 0 \\ &\equiv \langle \text{nt } 15 \rangle x \end{aligned}$$

stelling 18

$$\begin{aligned} x \wedge y &\equiv y \wedge x \\ &\equiv \langle \text{ax } 12 \rangle x \equiv y \equiv x \vee y \equiv y \equiv x \equiv y \vee x \\ &\equiv \langle \text{ax } 7, 9 \rangle x \equiv y \equiv x \vee y \equiv x \equiv y \equiv x \vee y \rightarrow \text{cont. van ax 2} \end{aligned}$$

stelling 19

zie currus p. 50

† stelling 20

$$\begin{aligned}
 x \wedge x &\equiv x & x, y &:= x, x \\
 x \wedge x &\equiv \langle ax 12 \rangle & x &\equiv x \equiv x \vee x \\
 &\equiv \langle ax 9 \rangle & x &\equiv x \equiv x \\
 &\equiv \langle st 4 \rangle & 1 &\equiv x & (y := x) \\
 &\equiv \langle st 4, \text{wafdenaar } 1 \rangle & x &
 \end{aligned}$$

stelling 21

currus p. 51

† stelling 22

$$\begin{aligned}
 x \wedge 0 &\equiv 0 \\
 x \wedge 0 &\equiv \langle ax 12 \rangle & x &\equiv 0 \equiv x \vee 0 \\
 &\equiv \langle st 15 \rangle & x &\equiv 0 \equiv x & (x \equiv x \equiv 0) \\
 &\equiv \langle ax 3 \rangle & 1 &\equiv 0 & (y := x) \\
 &\equiv \langle st 4 \rangle & 0 & & (y := 0)
 \end{aligned}$$

stelling 23

$$\begin{aligned}
 x \wedge (y \wedge z) &\equiv (x \wedge y) \wedge (x \wedge z) \\
 \downarrow \\
 (x \wedge y) \wedge (x \wedge z) &\equiv \langle st 18 \rangle & (y \wedge x) \wedge (x \wedge z) \\
 &\equiv \langle st 19 \rangle & y \wedge (x \wedge (x \wedge z)) & \text{stelling 18} \\
 &\equiv \langle st 20 \rangle & y \wedge (x \wedge z) \\
 &\equiv \langle st 18 \rangle & (y \wedge x) \wedge z \\
 &\equiv \langle st 18 \rangle & (x \wedge y) \wedge z \\
 &\equiv \langle st 19 \rangle & x \wedge (y \wedge z)
 \end{aligned}$$

stelling 24

currus p. 51 Linien

stelling 25 A)

$$\begin{aligned}
 x \wedge (x \vee y) &\equiv x \\
 &\equiv \langle ax 12 \rangle & x &\equiv (x \vee y) \equiv x \vee (x \vee y) \equiv x & \text{) distributiv ax 8} \\
 &\equiv \langle ax 9 \rangle & x &\equiv (x \vee y) \equiv (x \vee y) \equiv x \\
 &\rightarrow \text{invariantie van } ax 2 & y &:= (x \vee y)
 \end{aligned}$$

stelling 26 B)

$$\begin{aligned}
 x \vee (x \wedge y) &\equiv x \\
 &\equiv \langle ax 12 \rangle & x \wedge (x \wedge y) &\equiv x \equiv (x \wedge y) \equiv x & \text{) distrib. 16. 18} \\
 &\equiv \langle ax 9 \rangle & (x \wedge y) &\equiv x \equiv (x \wedge y) \equiv x \\
 &\rightarrow \text{invariantie v. } ax 2 & y &:= (x \wedge y) \text{ (wel symm.)}
 \end{aligned}$$

stelling 27

$$\begin{aligned}
 x \vee (y \wedge z) &\equiv (x \vee y) \wedge (x \vee z) \\
 (x \vee y) \wedge (x \vee z) &\equiv \langle ax 12 \rangle & (x \vee y) &\equiv (x \vee z) \equiv (x \vee y) \vee (x \vee z) \\
 &\equiv \langle st 16, ax 10 \rangle & x \vee (y \equiv z) &\equiv x \vee (y \vee z) \\
 &\equiv \langle ax 10 \rangle & x \vee ((y \equiv z) \equiv (y \vee z)) \\
 &\equiv \langle ax 12 \rangle & x \vee (y \wedge z)
 \end{aligned}$$

Stelling 28

$$x \wedge (y \vee z) \equiv (x \wedge y) \vee (x \wedge z)$$

$$(x \wedge y) \vee (x \wedge z) \equiv \langle ax12 \rangle (x \wedge y) \wedge (x \wedge z) \equiv (x \wedge y) \equiv \langle ax12 \rangle$$

$$\equiv \langle ax23 \rangle$$

$$x \wedge (y \vee z) \equiv (x \wedge y) \vee (x \wedge z)$$

$$(x \wedge y) \vee (x \wedge z) \equiv \langle ax27 \rangle ((x \wedge y) \vee x) \wedge ((x \wedge y) \vee z)$$

$$\equiv \langle ax25(b), ax2 \rangle x \wedge ((x \wedge y) \vee z)$$

$$\equiv \langle ax27, ax2 \rangle x \wedge ((z \vee x) \wedge (z \vee y))$$

$$\equiv \langle ax19 \rangle x \wedge ((x \vee z) \wedge (y \vee z))$$

$$\equiv \langle ax19, ax25(b) \rangle x \wedge (y \wedge z)$$

Stelling 28 A

belangrijke  
veelgebru. stelling!

$$\neg(x \wedge y) \equiv \neg x \vee \neg y$$

$$\neg(x \wedge y) \equiv \langle ax12 \rangle \neg x \equiv y \equiv x \vee \neg y$$

$$\equiv \langle ax17 \rangle \neg x \equiv y \vee \neg x$$

$$\equiv \langle ax6 \rangle \neg x \equiv \neg \neg y \vee \neg x$$

$$\equiv \langle ax17 \rangle \neg \neg y \vee \neg x$$

$$\forall x, y: x \equiv y$$

$$\forall x: x \equiv \neg \neg x$$

o Stelling 28 B

$$\neg(x \vee y) \equiv \neg x \wedge \neg y$$

$$\neg(x \vee y) \equiv \langle ax12 \rangle \neg(x \equiv y \equiv x \wedge y)$$

$$\equiv \langle ax5 \rangle x \equiv y \equiv \neg(x \wedge y)$$

$$\equiv \langle ax29A \rangle x \equiv y \equiv \neg x \vee \neg y$$

$$\equiv \langle ax12 \rangle x \equiv y \equiv \neg x \equiv \neg y \equiv \neg x \wedge \neg y$$

$$\equiv \langle ax13 \rangle 0 \equiv 0 \equiv \neg x \wedge \neg y$$

$$\equiv \langle ax3 \rangle 1 \equiv \neg x \wedge \neg y$$

$$\equiv \langle ax4 \rangle \neg x \wedge \neg y$$

$$y := 0$$

Stelling 30

$$x \wedge y \equiv x \wedge \neg \neg y \equiv \neg \neg x$$

$$x \wedge y \equiv x \wedge \neg \neg y \equiv \langle ax6 \rangle \neg \neg x \wedge \neg \neg y \equiv \neg \neg x \wedge \neg \neg y$$

$$\equiv \langle ax5 \rangle \neg(\neg x \wedge \neg y) \equiv \neg(\neg x \vee \neg y)$$

$$\equiv \langle ax5 \rangle \neg x \vee \neg y \equiv \neg x \vee \neg y$$

$$\equiv \langle ax4 \rangle \neg x \vee \neg y \equiv \neg x \equiv \neg x \vee \neg y$$

$$\equiv \langle ax4 \rangle \neg x$$

Stelling 31

curso p. 51

0 stelling 32

$$\begin{aligned}
x \wedge (y \equiv x) &\equiv x \wedge y \\
x \wedge (y \equiv x) &\equiv \langle ax12 \rangle x \equiv (y \equiv x) \equiv x \vee (y \equiv x) \\
&\equiv \langle ax10 \rangle x \equiv x \equiv y \equiv x \vee y \equiv x \vee x \\
&\equiv \langle ax9, ax3 \rangle 1 \equiv y \equiv x \vee y \equiv x \\
&\equiv \langle bk4, ax12 \rangle x \wedge y
\end{aligned}$$

0. stelling 33

$$\begin{aligned}
(x \equiv y) \wedge (z \equiv x) &\equiv (x \equiv y) \wedge (z \equiv y) \\
&\equiv \langle ax12 \rangle (x \equiv y) \equiv (z \equiv x) \equiv (x \equiv y) \vee (z \equiv x) \\
&\equiv (x \equiv y) \equiv (z \equiv y) \equiv (x \equiv y) \vee (z \equiv y) \\
&\equiv \langle ax3^{+14} \rangle z \equiv x \equiv z \equiv y \equiv (x \equiv y) \vee (z \equiv x) \equiv (x \equiv y) \vee (z \equiv y) \\
&\equiv \langle ax3 \rangle x \equiv y \equiv (x \equiv y) \vee (z \equiv x) \equiv (x \equiv y) \vee (z \equiv y) \\
&\equiv \langle ax10 \rangle x \equiv y \equiv (x \equiv y) \vee (z \equiv x \equiv z \equiv y) \\
&\equiv \langle ax3 \rangle x \equiv y \equiv (x \equiv y) \vee (x \equiv y) \\
&\equiv \langle ax9 \rangle x \equiv y \equiv x \equiv y \rightarrow ax2
\end{aligned}$$

stelling 34

$$\begin{aligned}
x \equiv y &\equiv (x \wedge y) \vee (\neg x \wedge \neg y) \\
(x \wedge y) \vee (\neg x \wedge \neg y) &\equiv \langle r17 \rangle (x \wedge y) \vee \neg(\neg x \wedge \neg y) \equiv (x \wedge y) \\
&\equiv \langle 25a \rangle (x \wedge y) \vee (x \vee \neg y) \equiv (x \wedge y) \\
&\equiv \langle ax7 \rangle x \vee y \vee (x \wedge y) \equiv (x \wedge y) \\
&\equiv \langle r25b \rangle x \vee y \equiv x \wedge y \\
&\equiv \langle ax12 \rangle x \equiv y
\end{aligned}$$

0 stelling 35

$$\begin{aligned}
x \not\equiv y &\equiv (\neg x \wedge y) \vee (x \wedge \neg y) \\
(\neg x \wedge y) \vee (x \wedge \neg y) &\equiv \langle r6 \rangle (\neg x \wedge y) \vee (\neg \neg x \wedge \neg y) \\
&\equiv \langle r34 \rangle \neg x \equiv y \\
&\equiv \langle r8 \rangle x \not\equiv y
\end{aligned}$$

stelling 36

$$\begin{aligned}
x \Rightarrow y &\equiv \neg x \vee y \\
&\equiv \langle ax13 \rangle x \vee y \equiv y \equiv \neg x \vee y \rightarrow \text{cont } r17
\end{aligned}$$

stelling 37

$$\begin{aligned}
x \Rightarrow y &\equiv x \wedge y \equiv x \\
&\equiv \langle ax13 \rangle x \vee y \equiv y \equiv x \wedge y \equiv x \\
&\equiv \langle ax12 \rangle x \wedge y \equiv y \equiv x \equiv x \equiv x \wedge y \equiv y \rightarrow \text{cont } ax2 \\
&\quad x, y := x \wedge y \equiv y, x
\end{aligned}$$

stelling 38

$$x \Rightarrow y \equiv \neg y \Rightarrow \neg x$$

$$\equiv \langle ax13, nt37 \rangle$$

$$x \vee y \equiv y \equiv \neg y \wedge \neg x \equiv \neg y$$

$$\equiv \langle nt29A \rangle x \vee y \equiv y \equiv \neg y \equiv \neg(x \vee y)$$

$$\equiv \langle nt3 \rangle 0 \equiv 0$$

$$\equiv \langle nt4 \rangle 1 \quad \square \quad y = 0$$

Stelling 39 zie cursus p. 52

Stelling 40

$$x \Rightarrow (y \Rightarrow z) \equiv x \Rightarrow y \equiv x \Rightarrow z$$

$$x \Rightarrow (y \Rightarrow z) \equiv \langle nt35 \rangle x \wedge y \equiv x \wedge z$$

$$\equiv \langle nt37 \rangle x \Rightarrow y \equiv x \equiv x \equiv x \Rightarrow z$$

$$\equiv \langle ax3 \rangle x \Rightarrow y \equiv x \Rightarrow z \equiv 1$$

$$\equiv \langle nt4 \rangle x \Rightarrow y \equiv x \Rightarrow z \quad \square$$

Stelling 41

$$x \Rightarrow (y \Rightarrow z) \equiv (x \Rightarrow y) \Rightarrow (x \Rightarrow z)$$

$$(x \Rightarrow y) \Rightarrow (x \Rightarrow z) \equiv \langle ax13 \rangle (x \Rightarrow y) \vee (x \Rightarrow z) \equiv (x \Rightarrow z)$$

$$\equiv \langle nt36 \rangle \{ \neg x \vee y \} \vee \{ \neg x \vee z \} \equiv \neg x \vee z$$

$$\equiv \langle ax9 \rangle (\neg x \vee z) \vee y \equiv \neg x \vee z$$

$$\equiv \langle nt17 \rangle (\neg x \vee z) \vee \neg y \equiv \neg x \vee z \equiv \neg x \vee z$$

$$\equiv \langle ax9 \rangle (\neg y \vee z) \vee \neg x$$

$$\equiv \langle ax13c \rangle (y \Rightarrow z) \vee \neg x \equiv \neg x \vee (y \Rightarrow z)$$

$$\equiv \langle nt36 \rangle x \Rightarrow (y \Rightarrow z) \quad \square$$

Stelling 42

$$x \wedge y \Rightarrow z \equiv x \Rightarrow (y \Rightarrow z)$$

$$x \Rightarrow (y \Rightarrow z) \equiv \langle nt37 \rangle x \Rightarrow (y \wedge z \equiv y)$$

$$\equiv \langle nt37 \rangle x \wedge (y \wedge z \equiv y) \equiv x$$

$$\equiv \langle nt37 \rangle x \wedge y \wedge z \equiv x \wedge y \equiv x \equiv x$$

$$\equiv \langle nt4 \rangle x \wedge y \wedge z \equiv x \wedge y$$

$$\equiv \langle nt37 \rangle x \wedge y \Rightarrow z \quad \square$$

○ Stelling 43

$$x \wedge (x \Rightarrow y) \equiv x \wedge y$$

$$x \wedge (x \Rightarrow y) \equiv \langle nt37 \rangle x \wedge (x \wedge y \equiv x)$$

$$\equiv \langle nt32 \rangle x \wedge x \wedge y$$

$$\equiv \langle nt20 \rangle x \wedge y \quad \square$$

○ Stelling 44

$$x \wedge (y \Rightarrow x) \equiv x$$

$$x \wedge (y \Rightarrow x) \equiv \langle nt38 \rangle x \wedge (\neg x \Rightarrow \neg y)$$

$$\equiv \langle nt37 \rangle x \wedge (\neg x \wedge \neg y \equiv \neg x)$$

$$\equiv \langle nt32 \rangle x \wedge \neg x \wedge \neg y \equiv \neg x \wedge x \equiv x$$

$$\equiv \langle 2x \text{ nt22} + \text{nt24} \rangle 0 \equiv 0 \equiv x$$

$$\equiv \langle \text{ax3} + \text{nt4} \rangle x$$

stelling 45

$$x \vee (x \Rightarrow y) \equiv 1$$

$$\begin{aligned}
x \vee (x \Rightarrow y) &\equiv \langle \text{r37} \rangle x \vee (x \wedge y') \equiv x \\
&\equiv \langle \text{ax10} \rangle x \vee (x \wedge y) \equiv x \vee x \\
&\equiv \langle \text{r53, ax9} \rangle x \equiv x \\
&\equiv \langle \text{ax3} \rangle 1 \quad \square
\end{aligned}$$

stelling 46

$$x \vee (y \Rightarrow x) \equiv y \Rightarrow x$$

$$\begin{aligned}
x \vee (y \Rightarrow x) &\equiv \langle \text{r38} \rangle x \vee (\neg x \Rightarrow y) \\
&\equiv \langle \text{r37} \rangle x \vee (\neg x \wedge y) \equiv \neg x \\
&\equiv \langle \text{ax10} \rangle x \vee (\neg x \wedge y) \equiv x \vee \neg x \\
&\equiv \langle \text{ax11, r4, r27} \rangle (x \vee \neg x) \wedge (x \vee y) \\
&\equiv \langle \text{ax11} \rangle 1 \wedge (x \vee y) \\
&\equiv \langle \text{r21} \rangle (x \vee y) \\
&\equiv \langle \text{r36} \rangle y \Rightarrow x \quad \square
\end{aligned}$$

stelling 47

$$x \vee y \Rightarrow x \wedge y \equiv x \equiv y$$

$$\begin{aligned}
x \vee y \Rightarrow x \wedge y &\equiv \langle \text{r37} \rangle (x \vee y) \wedge (x \wedge y) \equiv (x \vee y) \\
&\equiv \langle \text{r23} \rangle ((x \vee y) \wedge x) \wedge ((x \vee y) \wedge y) \equiv x \wedge y \\
&\equiv \langle \text{r25} \rangle x \wedge y \equiv x \vee y \\
&\equiv \langle \text{ax12} \rangle x \equiv y
\end{aligned}$$

stelling 48

$$x \Rightarrow x \equiv 1$$

$$\begin{aligned}
x \Rightarrow x &\equiv \langle \text{r36} \rangle \neg x \vee x \\
&\equiv \langle \text{ax9} \rangle 1
\end{aligned}$$

stelling 49

$$x \Rightarrow 1 \equiv 1$$

$$\begin{aligned}
x \Rightarrow 1 &\equiv \langle \text{r36} \rangle \neg x \vee 1 \\
&\equiv \langle \text{r14} \rangle 1
\end{aligned}$$

stelling 50

$$1 \Rightarrow x \equiv x$$

$$\begin{aligned}
1 \Rightarrow x &\equiv \langle \text{r36} \rangle \neg 1 \vee x \\
&\equiv \langle \text{ax4} \rangle 0 \vee x \\
&\equiv \langle \text{r22} \rangle x
\end{aligned}$$

stelling 51

$$x \Rightarrow 0 \equiv \neg x$$

$$\begin{aligned}
x \Rightarrow 0 &\equiv \langle \text{r36} \rangle \neg x \vee 0 \\
&\equiv \langle \text{r15} \rangle \neg x
\end{aligned}$$

stelling 52

$$0 \Rightarrow x \equiv 1$$

$$\begin{aligned}
0 \Rightarrow x &\equiv \langle \text{r36} \rangle \neg 0 \vee x \\
&\equiv \langle \text{r7} \rangle 1 \vee x \\
&\equiv \langle \text{r14} \rangle 1
\end{aligned}$$