

stelling 53A $x \Rightarrow x \vee y \equiv \langle \text{nt } 37 \rangle x \wedge (x \vee y) \equiv x$
 $(\equiv \langle \text{st } 25A \rangle x = x) \rightarrow \text{stelling } 25A$
 $(\equiv \langle \text{ax } 3 \rangle 1$

stelling 53B $x \wedge y \Rightarrow x \equiv \langle \text{nt } 37 \rangle x \wedge y \wedge x \equiv x \wedge y$
 $\equiv \langle \text{nt } 20 \rangle x \wedge y \equiv x \wedge y$
 $\equiv \langle \text{ax } 3 \rangle 1$

stelling 53C $x \wedge y \Rightarrow x \vee y \equiv \langle \text{nt } 37 \rangle (x \wedge y) \wedge (x \vee y) \equiv (x \wedge y)$
 $\equiv \langle \text{nt } 28 \rangle (x \wedge y \wedge x) \vee (x \wedge y \wedge y) \equiv (x \wedge y)$
 $\equiv \langle \text{nt } 20 \rangle (x \wedge y) \vee (x \wedge y) \equiv (x \wedge y)$
 $\equiv \langle \text{ax } 9 \rangle x \wedge y \equiv x \wedge y$
 $\equiv \langle \text{ax } 3 \rangle 1$

stelling 53D $x \vee (y \wedge z) \Rightarrow x \vee y \equiv \langle \text{nt } 27 \rangle (x \vee y) \wedge (x \vee z) \Rightarrow x \vee y$
 $\leftarrow \text{int } \text{nt } 53b$

stelling 53E $x \wedge y \Rightarrow x \wedge (y \vee z) \equiv \langle \text{nt } 28 \rangle x \wedge y \Rightarrow (x \wedge y) \vee (x \wedge z)$
 $\rightarrow \text{int } \text{nt } 53a$

stelling 54 $x \wedge (x \Rightarrow y) \Rightarrow y \equiv \langle \text{nt } 43 \rangle x \wedge y \Rightarrow y$
 $\equiv \langle \text{nt } 18 \rangle y \wedge x \Rightarrow y$
 $\rightarrow \text{int } \text{nt } 53b$

stelling 55 $(x \Rightarrow z) \wedge (y \Rightarrow z) \equiv (x \vee y) \Rightarrow z$
 $(x \Rightarrow z) \wedge (y \Rightarrow z) \equiv \langle \text{st } 36 \rangle (\neg x \vee z) \wedge (\neg y \vee z)$
 $\equiv \langle \text{nt } 27 \rangle z \vee (\neg x \wedge \neg y)$
 $\equiv \langle \text{nt } 29b \rangle z \vee \neg(x \vee y)$
 $\equiv \langle \text{st } 36 \rangle (x \vee y) \Rightarrow z$

stelling 56 $(x \Rightarrow z) \wedge (\neg x \Rightarrow z) \equiv z$
 $(x \Rightarrow z) \wedge (\neg x \Rightarrow z) \equiv \langle \text{st } 55 \rangle (x \vee \neg x) \Rightarrow z$
 $\equiv \langle \text{ax } 11 \rangle 1 \Rightarrow z$
 $\equiv \langle \text{st } 50 \rangle z$

stelling 57 $(x \Rightarrow y) \wedge (y \Rightarrow x) \equiv x \equiv y$
 $(x \Rightarrow y) \wedge (y \Rightarrow x) \equiv \langle \text{st } 38 \rangle (\neg x \vee y) \wedge (\neg y \vee x)$
 $\equiv \langle \text{ax } 12 \rangle (\neg x \vee y) \vee (\neg y \vee x) \equiv \neg x \vee y \vee \neg y \vee x$
 $\equiv \langle \text{ax } 3, \text{nt } 4, \text{ax } 11 \rangle \neg x \vee y \equiv \neg y \vee x$
 $\equiv \langle \text{nt } 6 + \text{nt } 29A \rangle \neg(x \wedge \neg y) \equiv \neg(y \wedge \neg x)$
 $\equiv \langle \text{nt } 5 \rangle x \wedge \neg y \equiv y \wedge \neg x$

$$\begin{aligned} &\equiv \langle \text{ax3} \rangle x \wedge y \equiv \neg x \equiv \neg x \wedge y \equiv \neg y \\ &\equiv \langle \text{ax3, ax4} \rangle \neg x \equiv \neg y \\ &\equiv \langle \text{ax5} \rangle x \equiv y \end{aligned}$$

stelling 58

$$\begin{aligned} &(x \Rightarrow y) \wedge (y \Rightarrow x) \Rightarrow (x \equiv y) \\ &\equiv \langle \text{ax5} \rangle (x \equiv y) \Rightarrow (x \equiv y) \\ &\equiv \langle \text{ax4} \rangle 1 \end{aligned}$$

stelling 59A

$$\begin{aligned} &((x \Rightarrow y) \wedge (y \Rightarrow z)) \Rightarrow (x \Rightarrow z) \\ &(x \Rightarrow y) \wedge (y \Rightarrow z) \Rightarrow \langle \text{ax3c} \rangle (x \Rightarrow y) \vee (y \Rightarrow z) \\ &\equiv \langle \text{ax3b} \rangle (\neg x \vee y \vee \neg y \vee z) \\ &\equiv \langle \text{ax11} \rangle (\neg x \vee z) \\ &\equiv \langle \text{ax3} \rangle x \Rightarrow z \quad \square \end{aligned}$$

stelling 59B

$$\begin{aligned} &(x \equiv y) \wedge (y \Rightarrow z) \Rightarrow (x \Rightarrow z) \\ &((x \equiv y) \wedge (y \Rightarrow z)) \equiv \langle \text{ax5} \rangle (x \Rightarrow y) \wedge (y \Rightarrow z) \wedge (y \Rightarrow z) \\ &\Rightarrow \langle \text{ax3b} \rangle (x \Rightarrow y) \wedge (y \Rightarrow z) \\ &\Rightarrow \langle \text{ax5A} \rangle x \Rightarrow z \end{aligned}$$

stelling 59C

$$\begin{aligned} &(x \Rightarrow y) \wedge (y \equiv z) \Rightarrow (x \Rightarrow z) \\ &\equiv \langle \text{ax5} \rangle (x \Rightarrow y) \wedge (y \Rightarrow z) \wedge (z \Rightarrow y) \\ &\Rightarrow \langle \text{ax3b} \rangle (x \Rightarrow y) \wedge (y \Rightarrow z) \\ &\Rightarrow \langle \text{ax59a} \rangle x \Rightarrow z \quad \square \end{aligned}$$

stelling 60A

$$(p \equiv q) \wedge \neg [v := p] \equiv (p \equiv q) \wedge \neg [v := q]$$

$$\begin{aligned} &\langle \text{ax15} \rangle \rightarrow (p \equiv q) \Rightarrow (\neg [v := p] \equiv \neg [v := q]) \\ &\equiv \langle \text{ax37} \rangle (p \equiv q) \wedge (\neg [v := p] \equiv \neg [v := q]) \equiv p \equiv q \\ &\equiv \langle \text{ax31} \rangle (p \equiv q) \wedge \neg [v := p] \equiv (p \equiv q) \wedge \neg [v := q] \\ &\equiv p \equiv q \equiv p \equiv q \\ &\equiv \langle \text{ax3} \rangle (p \equiv q) \wedge \neg [v := p] \equiv (p \equiv q) \wedge \neg [v := q] \equiv 1 \\ &\equiv \langle \text{ax4} \rangle (p \equiv q) \wedge \neg [v := p] \equiv (p \equiv q) \wedge \neg [v := q] \end{aligned}$$

Stelling 60 B

$$\begin{aligned}
 (p \equiv q) &\Rightarrow r[v := p] \equiv (p \equiv q) \Rightarrow r[v := q] \\
 &\equiv \langle \text{st 37} \rangle (p \equiv q) \wedge r[v := p] \equiv (p \equiv q) \wedge r[v := q] \equiv p \equiv q \\
 &\equiv p \equiv q \\
 &\equiv \langle \text{ax 3, st 4} \rangle p \equiv q \wedge r[v := p] \equiv (p \equiv q) \wedge r[v := q] \rightarrow \text{st 60A}
 \end{aligned}$$

Stelling 60 C

$$\begin{aligned}
 (\neg \wedge (p \equiv q)) &\Rightarrow r[v := p] \equiv (\neg \wedge (p \equiv q)) \Rightarrow r[v := q] \\
 &\equiv \langle \text{st 42} \rangle \neg \Rightarrow ((p \equiv q) \Rightarrow r[v := p]) \equiv \neg \Rightarrow ((p \equiv q) \Rightarrow r[v := q]) \\
 &\equiv \langle \text{st 40} \rangle \neg \Rightarrow (((p \equiv q) \Rightarrow r[v := p]) \equiv ((p \equiv q) \Rightarrow r[v := q])) \\
 &\equiv \langle \text{st 60b} \rangle \neg \Rightarrow 1 \\
 &\equiv \langle \text{st 49} \rangle 1 \quad \square
 \end{aligned}$$

↳ is een stelling dus mag je
gelijkstellen aan 1

Stelling 61 A

$$\begin{aligned}
 x &\Rightarrow r[v := x] \equiv x \Rightarrow r[v := 1] \\
 x &\Rightarrow r[v := x] \equiv \langle \text{st 4} \rangle (x \equiv 1) \Rightarrow r[v := x] \\
 &\equiv \langle \text{st 60b} \rangle (x \equiv 1) \Rightarrow r[v := 1] \\
 &\equiv \langle \text{st 4} \rangle x \Rightarrow r[v := 1] \quad \square
 \end{aligned}$$

Stelling 61 B
confer 60C

$$\begin{aligned}
 y \wedge x &\Rightarrow r[v := x] \equiv y \wedge x \Rightarrow r[v := 1] \\
 &\equiv \langle \text{st 42} \rangle y \Rightarrow (x \Rightarrow r[v := x]) \equiv y \Rightarrow (x \Rightarrow r[v := 1]) \\
 &\equiv \langle \text{st 40} \rangle y \Rightarrow (x \Rightarrow r[v := x]) \equiv y \Rightarrow r[v := 1] \\
 &\equiv \langle \text{st 61a, 49} \rangle 1 \quad \square
 \end{aligned}$$

Stelling 62 A

$$\begin{aligned}
 r[v := x] \Rightarrow x &\equiv r[v := 0] \Rightarrow x \\
 r[v := x] \Rightarrow x &\equiv \langle \text{st 38} \rangle \neg x \Rightarrow \neg r[v := x] \\
 &\equiv \langle \text{Sapp} \rangle \neg x \Rightarrow \neg r[v := x] \\
 &\equiv \langle \text{st 9} \rangle (x \equiv 0) \Rightarrow (\neg r)[v := x] \\
 &\equiv \langle \text{st 60(b)} \rangle (x \equiv 0) \Rightarrow (\neg r)[v := 0] \\
 &\equiv \langle \text{st 9, Sapp, st 38} \rangle r[v := 0] \Rightarrow x
 \end{aligned}$$

Stelling 62 B
confer 60C

$$\begin{aligned}
 r[v := x] \Rightarrow (x \vee y) &\equiv r[v := 0] \Rightarrow (x \vee y) \\
 &\equiv \langle \text{st 38} \rangle \neg(x \vee y) \Rightarrow \neg r[v := x] \equiv \neg(x \vee y) \Rightarrow \neg r[v := 0] \\
 &\equiv \langle \text{st 29b} \rangle \neg y \wedge \neg x \Rightarrow \neg r[v := x] \equiv \neg(y \wedge \neg x) \Rightarrow \neg r[v := 0] \\
 &\equiv \langle \text{Sapp, st 42} \rangle \neg y \Rightarrow (\neg x \Rightarrow (\neg r)[v := x]) \equiv \neg y \Rightarrow (\neg x \Rightarrow (\neg r)[v := 0]) \\
 &\equiv \langle \text{st 40} \rangle \neg y \Rightarrow [\neg x \Rightarrow (\neg r)[v := x]] \equiv \neg x \Rightarrow (\neg r)[v := 0] \\
 &\equiv \langle \text{Stepp, st 38, st 62A} \rangle \neg y \Rightarrow 1 \\
 &\equiv \langle \text{st 49} \rangle 1
 \end{aligned}$$